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To Develop an Deterministic Inventory Model using the Cubic Demand Rate and the Weibull Distribution as Deterioration Rate

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ABSTRACT: This research looks into an inventory model for decomposing items or products with a cubic demand over time, with an Weibull deterioration rate. In the model, shortages are allowed. It also shows that the cubic demand function is convex and yields the best result. A three-dimensional graphical representation of this model's convexity is shown. An illustration is created to double-check the model. The ideal solution was put through a sensitivity analysis with respect to the main parameters, and the results were presented.

KEYWORDS: Deterioration, cubic demand, Shortages, Total inventory cost.

I. INTRODUCTION

Inventory is defined as the stock of items kept on hand to ensure a trade or business's efficient and smooth operation. It also contributes to the company's growth. Manufacturers, schools, farms, hospitals, and higher education institutions all rely on it. Both merchants and wholesalers must keep their inventory of things or products at a minimum. It is made up of several diverging conditions that can be turned into models. These circumstances could include deterministic demand, demand changes over time, degradation, and so forth.

For many inventory products, like fashionable items, dairy goods, fruits and vegetables, and so on, the hypothesis of not changing demand rate is not appropriate; the demand rate may be dependent on time, stock, and on price. Items or things that deteriorate are referred to as degrading items. The inventory system is plagued by issues caused by the deterioration of items or products.

The change in demand rate could also be linear, i.e., demand might increase or decrease linearly with time. We employ a linear polynomial as a demand function for linear demand. It should occasionally be a significant change in demand, i.e., demand is rapidly increasing with relevance time.

An inventory mathematical model for degrading products with exponentially deteriorating demand was proposed by Liang- Yuh Ouyang, Kun-Shan WU, and Mei-Chuan CHENG (2005). Alfares (2007) looked at the inventory policy for an item with a demand rate dependent on the stock level and a storage time dependent on the holding cost. The case in which degrading rate follows Weibull distributions, C. K. Tripathy* and U. Mishra (2010) devised a listing model. A list model for decaying products was presented by R. Amutha and Dr. E. Chandrasekaran (2012). Demand was supposed to be a straight line in this model. The cost of holding is calculated as a linear function of time. Dr. Ravish Kumar Yadav and Ms. PratibhaYadav(2013) demonstrate the production model with assumption of cubic demand rate. Garima Sharma and Bhawna Vyas (2018) suggested a inventory model for decaying products with a Weibull distribution. Demand is measured to be a one degree function of time, with shortages permitted and partially backlogged. Ganesh Kumar, Sunita, and Ramesh Inaniyan (2020) proposed a listing model with a time-dependent demand rate employing various factors. It is supposed that the demand rate is a three degree polynomial of your time.

The demand rate is treated as a cubic polynomial of time in this paper, and the deterioration rate fluctuates with time. The cost of the order is believed to be constant and does not fluctuate over time. This model's convexity is checked using a three-dimensional graphical representation. To validate the model, an illustration is also created.

II. NORMS AND REPRESENTATIONS

The mathematical model is explained using the norms and representations listed below.

Norms

1. a, b, c, and d are constant.
2. The demand rate $f(t)$ at time t is supposed as $f(t) = a + bt + ct^2 + dt^3$; a, b, c, d is constant.
3. Replenishment takes place.
4. Shortages are permitted.
5. $\theta(t) = \alpha\beta t^{\beta-1}$ Denotes the deterioration rate.

Representations

C_{SC}	Shortage cost per unit per unit time.
C_{OC}	Ordering cost per order.
C_{DC}	Deterioration cost.
C_{HC}	Holding Cost.
W	The maximum inventory level for each ordering cycle.
S	Shortage level for each cycle.
Q	The order quantity ($Q = W + s$).
$Q(t)$	Inventory level at time t .
t_1	Time at which shortages start.
T	The total length of each ordering cycle.
TC	Total inventory cost over the period (0, T).
α	Weibull distribution parameter
β	Weibull distribution parameter

MATHEMATICAL FORMULATION:

The graph below (Figure 1) shows how inventory changes over time. The ideal order quantity, Q , and the total optimal inventory cost, TC , are shown in this diagram.

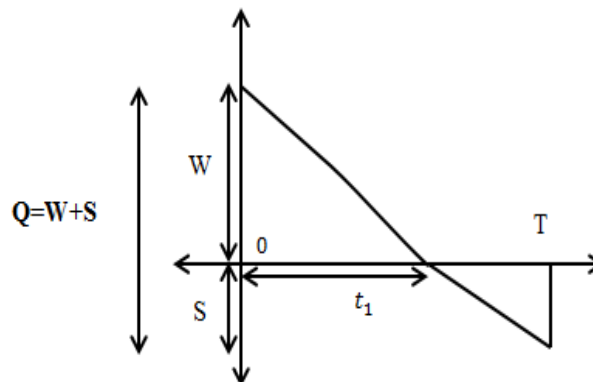


Figure 1: Inventory level (Q) vs time

At $t=0$, the inventory level reaches its extreme. Following that, the inventory level lowers over the time range $[0, t_1]$ until falling to zero at $t=t_1$. Further, at $t = t_1$, shortages occur during the time interval $[t_1, T]$.

Now till the shortages are allowed at interval $[0, t_1]$, the differential equation is given by:

$$\frac{dQ_1(t)}{dt} + \alpha\beta t^{\beta-1}Q_1(t) = -(a + bt + ct^2 + dt^3); 0 \leq t \leq t_1 \quad (1)$$

And during the interval $[t_1, T]$, the shortage occurs, so the differential equation is given by:-

$$\frac{dQ_2(t)}{dt} = -(a + bt + ct^2 + dt^3); t_1 \leq t \leq T \quad (2)$$

With the boundary conditions: $t=0$, $Q(0)=W$

$T=t_1$; $Q(t_1)=0$

$T=T$; $Q(T)=S$

Now, by solving the above equations (1), we get:

$$Q_1(t) = \left[a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{d}{4}(t_1^4 - t^4) + \frac{a\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1}) + \frac{b\alpha}{\beta+2}(t_1^{\beta+2} - t^{\beta+2}) + \frac{c\alpha}{\beta+3}(t_1^{\beta+3} - t^{\beta+3}) + \frac{d\alpha}{\beta+4}(t_1^{\beta+4} - t^{\beta+4}) - \left(a\alpha(t_1 t^\beta - t^{\beta+1}) + \frac{b\alpha}{2}(t_1^2 t^\beta - t^{\beta+2}) + \frac{c\alpha}{3}(t_1^3 t^\beta - t^{\beta+3}) + \frac{d\alpha}{4}(t_1^4 t^\beta - t^{\beta+4}) + \frac{a\alpha^2}{\beta+1}(t_1^{\beta+1} t^\beta - t^{2\beta+1}) + \frac{b\alpha^2}{\beta+2}(t_1^{\beta+2} t^\beta - t^{2\beta+2}) + \frac{c\alpha^2}{\beta+3}(t_1^{\beta+3} t^\beta - t^{2\beta+3}) + \frac{d\alpha^2}{\beta+4}(t_1^{\beta+4} t^\beta - t^{2\beta+4}) \right) \right] \quad (3)$$

By solving equation (2), we get:

$$Q_2(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) + \frac{d}{4}(t_1^4 - t^4) \quad (4)$$

Now, at $t=0$, the maximum inventory level for each cycle is given by:

$Q(0)=W$, $t=0$

$$W = Q_1(0) = at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \frac{dt_1^4}{4} + \frac{aat_1^{\beta+1}}{\beta+1} + \frac{bat_1^{\beta+2}}{\beta+2} + \frac{cat_1^{\beta+3}}{\beta+3} + \frac{dat_1^{\beta+4}}{\beta+4}$$

And at $t=T$, the maximum amount of cubic demand per cycle is given by:

$t=T$, $Q_2(t)=-S$

$$S = - \left[a(t_1 - T) + \frac{b}{2}(t_1^2 - T^2) + \frac{c}{3}(t_1^3 - T^3) + \frac{d}{4}(t_1^4 - T^4) \right]$$

Now, the order quantity per cycle is:

$$Q = W + S = at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + \frac{dt_1^4}{4} + \frac{aat_1^{\beta+1}}{\beta+1} + \frac{bat_1^{\beta+2}}{\beta+2} + \frac{cat_1^{\beta+3}}{\beta+3} + \frac{dat_1^{\beta+4}}{\beta+4} - \left[a(t_1 - T) + \frac{b}{2}(t_1^2 - T^2) + \frac{c}{3}(t_1^3 - T^3) + \frac{d}{4}(t_1^4 - T^4) \right] \quad (5)$$

Holding cost per unit per unit time is given by:

$$\text{Holding Cost per cycle} = C_{HC} \int_0^{t_1} Q_1(t) dt$$

$$\text{Holding Cost per cycle} = C_{HC} \left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4} + \frac{dt_1^5}{5} + \frac{a\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{b\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{c\alpha\beta t_1^{\beta+4}}{(\beta+1)(\beta+4)} + \frac{d\alpha\beta t_1^{\beta+5}}{(\beta+1)(\beta+5)} + \frac{a\alpha^2 t_1^{2\beta+2}}{(\beta+1)(2\beta+2)} + \frac{b\alpha^2 t_1^{2\beta+3}}{(\beta+1)(2\beta+3)} + \frac{c\alpha^2 t_1^{2\beta+4}}{(\beta+1)(2\beta+4)} + \frac{d\alpha^2 t_1^{2\beta+5}}{(\beta+1)(2\beta+5)} \right)$$

Shortages cost per unit per unit time is given by:

$$\text{Shortage Cost per cycle} = (-) C_{SC} \int_{t_1}^T Q_2(t) dt \quad (6)$$

$$\text{Shortage Cost per cycle} = -C_{SC} \left[a \left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2} \right) + b \left(\frac{t_1^2 T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3} \right) + c \left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4} \right) + d \left(\frac{t_1^4 T}{4} - \frac{T^5}{20} - \frac{t_1^5}{5} \right) \right]$$

Ordering cost per order is given by:

Ordering cost per order = C_{OC}

Now, the deteriorating cost is given by:

$$\text{Cost due to Deterioration} = C_{DC} \left[W - \int_0^{t_1} Q(t) dt \right]$$

$$\text{Cost due to Deterioration} = C_{DC} \left[\frac{aat_1^{\beta+1}}{\beta+1} + \frac{bat_1^{\beta+2}}{\beta+2} + \frac{cat_1^{\beta+3}}{\beta+3} + \frac{dat_1^{\beta+4}}{\beta+4} \right] \quad (7)$$

Therefore, the total cost per unit time per unit cycle is given by:

$$TC = \frac{1}{T} (\text{Holding Cost per cycle} + \text{Shortage Cost per cycle} + \text{Ordering cost per cycle} + \text{Cost due to Deterioration}) \quad (9)$$

$$TC = \frac{1}{T} \left\{ C_{HC} \left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4} + \frac{dt_1^5}{5} + \frac{a\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{b\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{c\alpha\beta t_1^{\beta+4}}{(\beta+1)(\beta+4)} + \frac{d\alpha\beta t_1^{\beta+5}}{(\beta+1)(\beta+5)} + \frac{a\alpha^2 t_1^{2\beta+2}}{(\beta+1)(2\beta+2)} + \frac{b\alpha^2 t_1^{2\beta+3}}{(\beta+1)(2\beta+3)} + \frac{c\alpha^2 t_1^{2\beta+4}}{(\beta+1)(2\beta+4)} + \frac{d\alpha^2 t_1^{2\beta+5}}{(\beta+1)(2\beta+5)} \right) - C_{SC} \left[a \left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2} \right) + b \left(\frac{t_1^2 T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3} \right) + c \left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4} \right) + d \left(\frac{t_1^4 T}{4} - \frac{T^5}{20} - \frac{t_1^5}{5} \right) \right] + C_{OC} + C_{DC} \left[\frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{\beta+2} + \frac{c\alpha t_1^{\beta+3}}{\beta+3} + \frac{d\alpha t_1^{\beta+4}}{\beta+4} \right] \right\}$$

This is the essential condition to minimize the total cost of inventory.

$$\frac{d(TC)}{dT} = -\frac{1}{T^2} \left\{ C_{HC} \left[\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{ct_1^4}{4} + \frac{dt_1^5}{5} + \frac{a\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{b\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{c\alpha\beta t_1^{\beta+4}}{(\beta+1)(\beta+4)} + \frac{d\alpha\beta t_1^{\beta+5}}{(\beta+1)(\beta+5)} + \frac{a\alpha^2 t_1^{2\beta+2}}{(\beta+1)(2\beta+2)} + \frac{b\alpha^2 t_1^{2\beta+3}}{(\beta+1)(2\beta+3)} + \frac{c\alpha^2 t_1^{2\beta+4}}{(\beta+1)(2\beta+4)} + \frac{d\alpha^2 t_1^{2\beta+5}}{(\beta+1)(2\beta+5)} \right] - C_{SC} \left[a \left(t_1 T - \frac{T^2}{2} - t_1 \right) + b \left(\frac{t_1^2 T}{2} - \frac{T^3}{6} - \frac{t_1^3}{3} \right) + c \left(\frac{t_1^3 T}{3} - \frac{T^4}{12} - \frac{t_1^4}{4} \right) + d \left(\frac{t_1^4 T}{4} - \frac{T^5}{20} - \frac{t_1^5}{5} \right) \right] + C_{OC} + C_{DC} \left[\frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{\beta+2} + \frac{c\alpha t_1^{\beta+3}}{\beta+3} + \frac{d\alpha t_1^{\beta+4}}{\beta+4} \right] \right\} - \frac{1}{T} \left\{ C_{SC} \left[a(t-T) + b \left(\frac{t^2}{2} - \frac{T^2}{2} \right) + c \left(\frac{t^3}{3} - \frac{T^3}{3} \right) + d \left(\frac{t^4}{4} - \frac{T^4}{4} \right) \right] \right\} \quad (10)$$

$$\frac{d(TC)}{dt_1} = \frac{1}{T} \left\{ C_{HC} \left(at_1 + bt_1^2 + ct_1^3 + dt_1^4 + \frac{a\alpha\beta t_1^{\beta+2}}{(\beta+1)t_1} + \frac{b\alpha\beta t_1^{\beta+3}}{(\beta+1)t_1} + \frac{c\alpha\beta t_1^{\beta+4}}{(\beta+1)t_1} + \frac{d\alpha\beta t_1^{\beta+5}}{(\beta+1)t_1} - \frac{a\alpha^2 t_1^{2\beta+2}}{(\beta+1)t_1} - \frac{b\alpha^2 t_1^{2\beta+3}}{(\beta+1)t_1} - \frac{c\alpha^2 t_1^{2\beta+4}}{(\beta+1)t_1} - \frac{d\alpha^2 t_1^{2\beta+5}}{(\beta+1)t_1} \right) - C_{SC} [a(T-t_1) + b(t_1 T - t_1^2) + (t_1^2 T - t_1^3) + d(t_1^3 T - t_1^4)] + C_{DC} \left[\frac{a\alpha t_1^{\beta+1}}{t_1} + \frac{b\alpha t_1^{\beta+2}}{t_1} + \frac{c\alpha t_1^{\beta+3}}{t_1} + \frac{d\alpha t_1^{\beta+4}}{t_1} \right] \right\} \quad (11)$$

We get the optimal values of t_1 and T by solving equations (11) & (12) by using MAPLE 15.

(12)

NUMERICAL ILLUSTRATION:

Now check the optimality of the model with the help of numerical illustration and solve the illustration with the help of Maple 15. To explain the model numerically, assume the following parameters of the inventory system are:

$$a = 10, b = 4, c = 4, d = 3, C_{HC} = 4, C_{SC} = 15, C_{OC} = 100, C_{DC} = 10, \alpha = 0.1, \beta = 0.5$$

Under the above-given parameters, by using Maple 15 get the optimal shortage value $t_1 = 1.234454529$ per unit time and the optimal length of the ordering cycle is $T = 1.559308506$ unit time. The total inventory cost is $TC = 176.8809149$

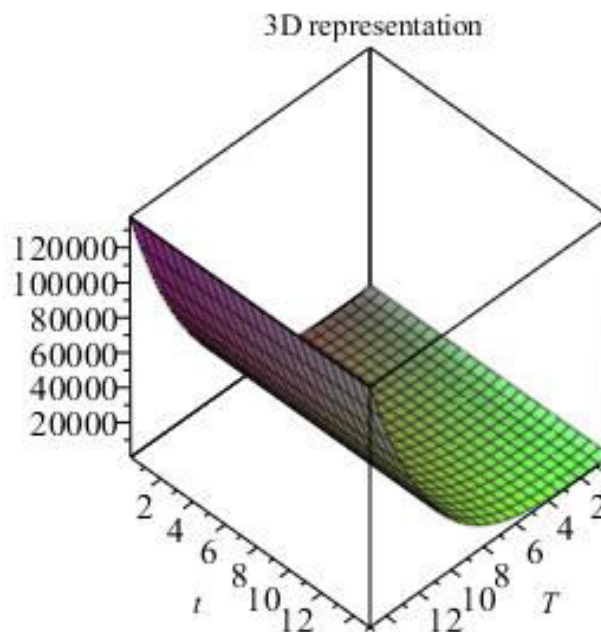


Figure 2: 3-Dimensional Graphical representation of TC of Inventory Model

**SENSITIVITY ANALYSIS:**

Here we study the effect of changes in the model parameters such that $a, b, c, d, C_{HS}, C_{CS}, C_{OC}, C_{DC}, \alpha$, and β . The outcome is given in the below table:

Parameter	% change	Change in			
		T	t_1	Q	TC
a	+20%	1.603896271	1.289146952	36.81623890	191.8371917
	+10%	1.568618139	1.216711017	32.56914302	180.8997758
	-10%	1.537839289	1.208227222	29.04572767	169.0347097
	-20%	1.517034484	1.182951026	26.64390636	161.2846331
b	+20%	1.539550707	1.212124022	31.74566643	178.8959541
	+10%	1.549380965	1.223202067	31.64593728	177.8119517
	-10%	1.569331489	1.245882449	31.43124085	175.5993973
	-20%	1.579447871	1.257486732	31.31578652	174.4701583
c	+20%	1.529580785	1.201760482	31.35119014	178.9090809
	+10%	1.544189173	1.217788367	31.44598056	177.8277506
	-10%	1.574966130	1.251800094	31.63662222	175.5633731
	-20%	1.591191112	1.269869746	31.73191830	174.3759446
d	+20%	1.513545373	1.186213620	30.54994917	179.3761816
	+10%	1.535509911	1.209408379	31.45575446	179.3439794
	-10%	1.59875912	1.28756974	32.11259704	175.2869791
	-20%	1.613746592	1.291436610	32.74815908	173.7565270
C_{HC}	+20%	1.479911573	1.091073046	28.31101181	183.1714517
	+10%	1.518505997	1.161619901	29.84438202	180.1794500
	-10%	1.603096218	1.310575724	33.45177555	172.7003266

	-20%	1.650807746	1.391122643	35.64302355	168.0487238
C_{SC}	+20%	1.604687345	1.347707128	33.63994747	188.8085173
	+10%	1.581764910	1.293568251	32.57225675	182.9825936
	-10%	1.537901867	1.169430727	30.56672565	169.9449504
	-20%	1.518470657	1.097310232	29.67942154	162.6079893
C_{OC}	+20%	1.611818504	1.284955997	33.71077833	189.3228638
	+10%	1.586564268	1.260866443	32.65301787	183.0701644
	-10%	1.529580629	1.205109266	30.36288365	170.2391594
	-20%	1.496687219	1.171910476	29.09960476	163.6314152
C_{DC}	+20%	1.542710635	1.203614726	30.83734305	178.7133475
	+10%	1.551012432	1.219072723	31.18768414	177.7250143
	-10%	1.567605704	1.249771493	31.89822821	175.6776823
	-20%	1.575910508	1.265034184	32.25905017	174.6181473
α	+20%	1.541152022	1.200863436	31.07218564	178.8350784
	+10%	1.550191264	1.217632503	31.30781452	177.7900824
	-10%	1.568517381	1.251349728	31.77267919	175.6034615
	-20%	1.577831636	1.268338043	32.00252961	174.4597498
β	+20%	1.552778559	1.224758923	31.22345685	176.5596111
	+10%	1.559874563	1.234868441	31.58745964	176.6569845
	-10%	1.562762624	1.239576280	31.71076676	176.7933240
	-20%	1.566354255	1.244897028	31.88819454	176.8757253

- ❖ With the increase in a, b, d; TC and Q increase..
- ❖ With the increase in c, TC increases and Q decreases.
- ❖ If C_{HC} (Holding Cost) and C_{DC} (Deterioration Cost), increases, TC increases, and Q will decrease.
- ❖ If C_{SC} (Shortage Cost), C_{OC} (Ordering Cost) increases, Q and TC will increase.
- ❖ If α , β increases, Q decreases.

III. CONCLUSION

The sensitivity analysis found that in the case of cubic demand (a function of time), the model predicts that the rate of deterioration will alter over time. It can be shown that parameters a and b are directly proportional to TC, whereas the parameters c, is inversely proportional to TC. It also demonstrates that this technique can be used to determine the total inventory cost.. Finally, a graphical depiction is used to verify the model. The obtained findings determine the model's stability. This model can be expanded or replaced with a different demand rate.



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