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# Nano Pre $\alpha^*$ - Closed Set in Nano Topological Space

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**ABSTRACT:** In this paper we have introduced a new class of nano closed set named as nano pre  $\alpha^*$  - closed set in nano topological spaces. We have investigated their properties. Also we have analyzed the relationship between nano pre  $\alpha^*$  - closed set with the existing sets in nano topological space.

**KEYWORDS:** Nano pre  $\alpha^*$  - closed

## I. INTRODUCTION

Nano topology was explored by M.Lellish Thivagar et.al can be described as a collection of nano approximations, a non-empty finite universe and empty set for which equivalence classes are building blocks. He introduced a nano topological space it respect to a subset  $X$  of an universe which is defined in terms of lower and upper approximations of  $X$ . The elements of a nano topological space are called the nano open sets. He also introduced nano closure and nano interior of a set. He also established the forms of nano-open sets such as nano  $\alpha$  - open sets, nano semi -open sets, nano regular open sets and nano pre-open sets in topological spaces.

## II. PRELIMINARIES

In this section, we recall some basic definitions.

**Definition 2.1[2]:** Let  $U$  be a non- empty finite set of objects is called the universe and  $R$  be an equivalence relation on  $U$  named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair  $(U, R)$  is said to be the approximation space. Let  $X \subseteq U$ .

- (i) The lower approximation of  $X$  with respect to  $R$  is the set of all objects, which can be for certain classified as  $X$  with respect to  $R$  and it is denoted by  $L_R(X)$ . That is  $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$  where  $R(x)$  denotes the equivalence class determined by  $x$ .
- (ii) The upper approximation of  $X$  with respect to  $R$  is the set of all objects, which can be possibly classified as  $X$  with respect to  $R$  and it is denoted by  $U_R(X)$ . That is  $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$
- (iii) The boundary region of  $X$  with respect to  $R$  is the set of all objects, which can be classified neither as  $X$  nor as not-  $X$  with respect to  $R$  and it is denoted by  $B_R(X)$ . That is,  $B_R(X) = U_R(X) - L_R(X)$ .

**Definition 2.2[2]:** Let  $U$  be the universe,  $R$  be an equivalence relation on  $U$  and  $\tau_R(X) = \{\emptyset, U, L_R(X), U_R(X), B_R(X)\}$  where  $X \subseteq U$ . Then  $\tau_R(X)$  satisfies the following axioms:

- (i)  $U$  and  $\emptyset$  are in  $\tau_R(X)$ .
- (ii) The union of the elements of any sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .
- (iii) The intersection of the elements of any finite sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$

That is,  $\tau_R(X)$  is a topology on  $U$  called the nano topology on  $U$  with respect to  $X$ . The space  $(U, \tau_R(X))$  is called as nano topological space. The elements of  $\tau_R(X)$  are called nano – open sets of  $(U, \tau_R(X))$ .

**Remark 2.3 [2]** : If  $\tau_R(X)$  is the nano topology on  $U$  with respect to  $X$ , then the set  $\beta_R(X) = \{U, L_R(X), B_R(X)\}$  is the basis for  $\tau_R(X)$ .

**Definition 2.4 [2]**: If  $(U, \tau_R(X))$  is a nano topological space with respect to  $X$  where  $X \subseteq U$  and if  $A \subseteq U$ , then the nano interior of  $A$  is defined as the union of all nano – open subsets of  $A$  and it is denoted by  $Nint(A)$ . That is  $Nint(A)$  is the largest nano- open subset of  $A$ . The nano closure of  $A$  is defined as the intersection of all nano closed sets containing  $A$  and it is denoted by  $Ncl(A)$ . That is,  $Ncl(A)$  is the smallest nano closed set containing  $A$ .

**Definition 2.5 [2]**: If  $(U, \tau_R(X))$  is a nano topological space with respect to  $X$  with  $A \subseteq U$ . Then  $A$  is said to be a

- (i) nano  $\alpha$ - open if  $A \subseteq Nint(Ncl(Nint(A)))$ .
- (ii) nano regular open if  $A = Nint(Ncl(A))$ .

**Definition 2.6 [1]**: A subset  $H$  of a nano topological space  $(U, \tau_R(X))$  is called a nano semi generalized closed set ( nsg – closed ) if  $Nscl(H) \subseteq G$ , whenever  $H \subseteq G$  and  $G$  is nano semi - open set in  $(U, \tau_R(X))$ .

**Definition 2.7** : A subset  $A$  of  $(U, \tau_R(X))$  is called a nano  $\alpha^*$ - set if  $Nint(A) = Nint(Ncl(Nint(A)))$

**Example 2.8**: Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{c\}, \{b, d\}\}$  and let  $X = \{a, b\}$ . Then  $\tau_R(X) = \{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$  is a nano topology on  $U$  with respect to  $X$  and  $\tau_R^c(X) = \{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$ . A subset  $H$  of a nano topological space  $(U, \tau_R(X))$  is called a

- i) nano  $\alpha$  - closed :  $\{\phi, U, \{c\}, \{a, c\}, \{b, c, d\}\}$
- ii) nano regular – closed:  $\{\phi, U, \{a, c\}, \{b, c, d\}\}$
- iii) nano sg – closed:  $\{\phi, U, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$
- iv) nano  $\alpha^*$  - set:  $\{\phi, U, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$

### III. NANO PRE $\alpha^*$ -CLOSED SET IN NANO TOPOLOGICAL SPACE

In this section, we have introduced a new class of nano closed set called as nano pre  $\alpha^*$  - closed set in nano topological spaces and also analyzed its properties.

**Definition 3.1**: A subset  $A$  of  $(U, \tau_R(X))$  is called a nano pre  $\alpha^*$  - closed set (briefly  $Np\alpha^*$  - closed ) if  $Npcl(A) \subseteq G$  whenever  $A \subseteq G$  and  $G$  is  $N\alpha^*$  - set.

**Example 3.2**: Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{c\}, \{b, d\}\}$  and let  $X = \{a, b\}$ . Then  $\tau_R(X) = \{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$  is a nano topology on  $U$  and  $Np\alpha^*$  - closed sets are  $\{\phi, U, \{b\}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ .

**Theorem 3.3**: The intersection of two  $Np\alpha^*$ - closed sets are also a  $Np\alpha^*$ - closed set

**Proof**: Let  $A$  and  $B$  be  $Np\alpha^*$ - closed sets in  $(U, \tau_R(X))$ . Let  $V$  be a  $Np\alpha^*$  - closed set in  $(U, \tau_R(X))$  such that  $A \cap B \subseteq V$ . Then  $A \subseteq V$  and  $B \subseteq V$ . Since  $A$  and  $B$  are  $Np\alpha^*$ - closed,  $Npcl(A) \subseteq V$  and  $Npcl(B) \subseteq V$ . Hence  $Npcl(A \cap B) \subseteq Npcl(A) \cap Npcl(B) \subseteq V$ . Therefore  $A \cap B$  is  $Np\alpha^*$ - closed set in  $(U, \tau_R(X))$ .

**Remark 3.4**: The union of two  $Np\alpha^*$  - closed sets need not be a  $Np\alpha^*$ -closed set as seen from the following Example.



**Example 3.5:** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{c\}, \{b, d\}\}$  and let  $X = \{a, b\}$ . Then  $\tau_R(X) = \{\emptyset, U, \{a\}, \{a, b, d\}, \{b, d\}\}$  is a nano topology on  $U$  with respect to  $X$ . In Example 3.2,  $\{b\}$  and  $\{d\}$  are  $N\alpha^*$ -closed set but their union  $\{b, d\}$  is not a  $N\alpha^*$ -closed set.

**Theorem 3.6 :** Every nano closed set is  $N\alpha^*$ -closed set in nano topological spaces.

**Proof:** Let  $A$  be a Nano closed set and so  $G$  is a  $N\alpha^*$ -set. Since  $G$  is nano closed set,  $Ncl(A) \subseteq G$ . Since  $Npcl(G) \subseteq Ncl(G)$ , we have  $Npcl(G) \subseteq G$  where  $G$  is  $N\alpha^*$ -set.

Converse of the above theorem need not be true as seen from the following example.

**Example 3.7:** In Example 3.2,  $\{b\}$  and  $\{d\}$  are  $N\alpha^*$ -closed sets but not nano closed sets.

**Theorem 3.8 :** (i) Every  $Nr$  closed set is  $N\alpha^*$ -closed set in nano topological spaces.

(ii) Every  $N\alpha$ -closed set is  $N\alpha^*$ -closed set in nano topological spaces.

**Proof :** Proof follows as same as previous theorem .

Converse of the above theorem need not be true as seen from the following example.

**Example 3.9:** In Example 3.2,  $\{c\}$  and  $\{b, c\}$  are  $N\alpha^*$ -closed set but not a  $Nr$ -closed set.

**Example 3.10 :** In Example 3.2,  $\{c\}$  and  $\{a, b, c\}$  are  $N\alpha^*$ -closed set but not a  $N\alpha$ -closed set.

**Theorem 3.11:** Every  $N\alpha^*$ -closed set is  $Nsg$ -closed set.

**Proof:** Assume that  $A$  is  $N\alpha^*$ -closed set in  $(U, \tau_R(X))$ . Let  $A \subseteq V$ , where  $V$  is nano  $\alpha^*$ -set. Since  $A$  is  $N\alpha^*$ -closed set and every nano semi open set is nano  $\alpha^*$ -set, therefore  $Npcl(A) \subseteq V$  where  $V$  is nano semi-open set. Hence  $A$  is nano  $sg$ -closed set in  $(U, \tau_R(X))$ .

Converse of the above theorem need not be true as seen from the following example

**Example 3.12 :** In Example 2.8,  $\{a\}$  and  $\{b, d\}$  are  $Nsg$ -closed but not a  $N\alpha^*$ -closed set.

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