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Existence of a Relation for Finding All Prime Numbers

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ABSTRACT: Prime Numbers have always been a fascinating area in mathematics. Many properties of prime numbers are still not known and are being researched upon. One of those topics is the existence of a relation to find all the prime numbers. There are many relations which can define prime numbers but none of them is complete in itself. Every relation existing today has some drawbacks.

Hence, in this paper, we are going to look into certain relations that help in finding the prime numbers.

KEYWORDS: Prime Number, Relation, Factors, Greatest common divisor, Recurrence Relation, Function.

I. INTRODUCTION

A prime number is a number which is divisible by 1 or by itself i.e. it has no other divisors in the number system. So we can say that any prime number p has only two factors which are 1 and p itself. Any number which does not follow this rule is known as a composite number. A number q is said to be composite if its prime factorization contains a number q' which is neither equal to 1 nor equal to q itself.

One of the major concerns with respect to prime numbers is the occurrence of a relation or a function which can give us all the prime numbers that exist.

A relation from a set A to a set B is a set of ordered pairs (x, y) such that $x \in A$ and $y \in B$ and x is related to y by certain rule.

A function from set A to set B is a special type of relation in which every value of $x \in A$ has a unique value $y \in B$.

The major requirements for a relation to be a function are:

1. Every element in domain has a unique image in co-domain i.e. for $A \rightarrow B$, every $x \in A$ has a unique $y \in B$.
2. Every element in co-domain must have a pre-image in the domain i.e. for $A \rightarrow B$, every $y \in B$ must have a pre-image $x \in A$.

II. PROPERTIES OF PRIME NUMBERS

There are many properties which can be observed in prime numbers of which some are listed below:

1. The even number 2 is the only even number which is prime. All the other existing numbers known to be prime are odd.
2. Wilson's Theorem^[1]: Given a prime number p , $(p-1)!+1$ is divisible by p . This can also be stated as:
$$(p-1)! \equiv -1 \pmod{p}$$
3. Number of primes^[2]: The pi function, denoted by $\pi(x)$, gives the number of primes less than or equal to x . For e.g. $\pi(10)=4$ as the primes less than 10 are 2,3,5,7.
4. Goldbach's Conjecture^[3]: Also known as Goldbach Partition, it states that, every even integer greater than 2 can be expressed as the sum of two primes.



For e.g. $6=3+3$, $12=5+7$, etc.

The proof for the above conjecture isn't available till date.

Rowland's Formula for Primes

The formula given by Eric S. Rowland can be termed as one of the relations that can be used to find the primes. The formula is more of a recurrence relation which is given as:

$$a_n = a_{n-1} + \gcd(n, a_{n-1}), \text{ where } a_1=7^{[4]}$$

This formula gives first few numbers as 7, 8, 9, 10, 15, 18, 19, 20, 21, 22, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, and 69. The actual prime generator from the above sequence is the difference between two consecutive terms in the above sequence. It can be observed, the difference would either contain 1 or a prime number. The first few resulting differences from above sequence are 1, 1, 1, 5, 3, 1, 1, 1, 1, 11, 3, 1, 1, 1, 1, 1, 1, 1, 1, and 23.

If we remove the repetitive 1's the leftover numbers are prime numbers. If we even remove the repetitions as we can see that 3 is being repeated twice in the above sequence. The few primes that we get are 5, 3, 11, 23, 47, 101, 7, 13, 233, and so on.

The major disadvantage of this method includes:

1. The prime numbers are in a jumbled form and have multiple occurrences, i.e. a clear one-to-one correspondence can't be drawn so as to mark prime numbers.
2. The method is time consuming as many a times we come across the value '1' which needs to be discarded.

Hence, the above method isn't suitable for our purpose of finding the prime numbers.

$$P(n) = n^2 + n + 41 \quad \forall n \in \mathbb{N}$$

The relation written above has been a lot into discussion under the topic of induction. If we try to find the first few values from the above relation we get,

$$\begin{aligned} P(1) &= 43 \\ P(2) &= 47 \\ P(3) &= 53 \\ P(4) &= 61 \\ P(5) &= 71 \\ P(6) &= 83 \\ &\dots \text{ and so on} \end{aligned}$$

This pattern is followed upto $n=39$, i.e. we get prime numbers till $n=39$ where $P(n)=1601$ which is a prime number.

For $n=40$,

$$\begin{aligned} P(40) &= 40^2 + 40 + 41 \\ &= 40(40 + 1) + 41 \\ &= 40(41) + 41 \\ &= 41(41) \\ &= 41^2 = 1681, \end{aligned}$$

which is divisible by 41 apart from 1 and the number itself i.e. 1681.

Hence the formula breaks at $n=40$. Similarly there are many more values of n for which the relation breaks. For e.g. For $n=41$, $P(n)=1763$ which is also not a prime number.

Another disadvantage of the above formula is we can never get the prime numbers less than 43, hence the rest of the primes which are less than 43 will not be considered in the relation given.

The form $6n \pm 1 \quad \forall n \in \mathbb{N}$

The above form, originally, is stated as any prime number which is greater than 3 can be written either as $6n-1$ or $6n+1$ where $n \in \mathbb{N}$.^[5] Let us try to find our way to the above result.



According to the Euclid's Division Lemma

dividend = divisor \times quotient + remainder

So, for value of divisor equal to 6, we have $d = 6q + r$.

Now the possible values of r are 0, 1, 2, 3, 4, 5.

For $r=0$,

$$d = 6q$$

which can be rewritten as

$$d = 2(3q)$$

which is divisible by 2, hence composite.

For $r=2$,

$$d = 6q + 2$$

which can be rewritten as

$$d = 2(3q + 1)$$

which is divisible by 2, hence composite.

For $r=3$,

$$d = 6q + 3$$

which can be rewritten as

$$d = 3(2q + 1)$$

which is divisible by 3, hence composite.

The case where $r=4$, is similar as $r=2$ hence it will be divisible by 2 giving a composite number.

For $r=1$,

$$d = 6q + 1$$

And, for $r=5$,

$$d = 6q + 5$$

$$d = 6q + 6 - 1$$

$$d = 6(q + 1) - 1$$

$$d = 6q' - 1, \text{ where } q' = q + 1$$

Hence, we get primes for $6n \pm 1$.

But this form assures that every prime number can be written in this form but for every value of n we shall not get a prime number. For e.g. if $n=4$, $6n-1=23$ which is prime but $6n+1=25$ which is not a prime number.

Hence, it simply says that every prime number can be written as $6n \pm 1$ but not every value of n shall give a prime number using this formula.

Sieve of Eratosthenes

This method was introduced by the Greeks.

The method works as follows^[6]:

- Write all numbers from 2 till where the primes are required.
- The first number we encounter is 2, hence mark out all the multiples of 2 except 2 itself.
- Next number we encounter is 3, mark out all multiples of 3 except for 3 itself.
- The next number we encounter is 4 but it is marked as it is a multiple of 4.
- Now the next number is 5, mark out all the multiples of 5 except for 5 itself.
- Continue this till all the numbers are marked except for once which are the first multiples being encountered.



After doing all the above steps, the list of unmarked numbers are the list of the prime numbers.

The above method is till date the most efficient method known to find all the prime numbers upto a certain number in the real number system.

The biggest disadvantage of the above method is that it is very lengthy and time consuming to find a certain number is prime or not. This is because of the use of the method of elimination and not mapping particular prime numbers to certain key-pair values.

III. CONCLUSION

After examining all the above methods trying to find the prime numbers, we conclude that all the methods we have till date can either find the prime numbers upto some extent or can help in representing primes in certain forms.

But till date, we don't have any such method to map a one-to-one correspondence to the prime numbers.

Also from the above methods we also imply that we are very far away from finding such a one-to-one correspondence for representing primes.

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