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# The Matching Equivalent Classes of

$$P_2 \cup I_6 \cup T(1,1,n)$$

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**ABSTRACT:** Completely characterize the matching equivalent classes of  $P_2 \cup I_6 \cup T(1,1,n)$ , by the property of graph's matching polynomial and its maximum roots.

**KEYWORDS:** Matching polynomial, Matching equivalence, Matching uniqueness, The maximum real roots.

## I. INTRODUCTION

All graphs considered in this paper are undirected and simple (i.e., loops and multiple edges are not allowed). Let  $G = (V(G), E(G))$  be a graph with vertex set  $V(G) = \{v_1, v_2, \dots, v_n\}$  and edge set  $E(G) = \{e_1, e_2, \dots, e_m\}$ , where  $|V(G)| = n$  is the order and  $|E(G)| = m$  is the size of  $G$ . A spanning subgraph  $H$  is called a matching of  $G$ , if every connected component of  $H$  is isolated edge or isolated vertex.  $k$ -matching of  $G$  is a matching with  $k$  edges. In [1] E. J. Farrell denote the matching polynomial as

$$\mu(G, x) = \sum_{k \geq 0} (-1)^k p(G, k) x^{n-2k},$$

where  $p(G, k)$  is the number of  $k$ -matchings of  $G$ .

Two graphs  $G$  and  $H$  are called matching-equivalent if  $\mu(G, x) = \mu(H, x)$ , and denoted by  $G \sim H$ . The disjoint union of two graphs  $G$  and  $H$ , denoted by  $G \cup H$ , is the graph with vertex set  $V(G) \cup V(H)$  and edge set  $E(G) \cup E(H)$ .  $kG$  denotes the disjoint union of  $k$  copies of  $G$ . Let  $M(G)$  be the largest matching root of  $\mu(G, x)$ . We denote by  $P_n$  ( $n \geq 1$ ),  $C_n$  ( $n \geq 3$ ) a path and a cycle of order  $n$ , respectively. A graph  $D_{m,n}$  ( $m \geq 3, n \geq 1$ ) is defined as the graph obtained by identifying one end of the path  $P_{n+1}$  with a vertex of the cycle  $C_m$ . By  $T(a, b, c)$  denote the tree which has one 3-degree vertex  $u$  and three 1-degree vertices  $v_1, v_2, v_3$  and the distance between  $u$  and  $v_1, v_2, v_3$  are  $a, b, c$ , respectively. Let  $P_{n-2}$  be a path with vertices sequence  $1, 2, \dots, n-2$ ,  $I_n$  ( $n \geq 6$ ) denotes the tree obtained by adding pendant edges at vertices 2 and  $n-3$  of  $P_{n-2}$ , respectively. For a graph  $G$ , let  $G^c$  be the complement of  $G$ . The classes of matching equivalent graphs determined by  $G$  under  $\sim$  is denoted by  $[G]$ . In this paper, we Completely characterize the matching equivalent classes of  $P_2 \cup I_6 \cup T(1,1,n)$ .

## II. PRELIMINARIES

**Lemma 2.1**[1] Let  $G$  be a graph with  $k$  components  $G_1, G_2, \dots, G_k$ . Then  $\mu(G, x) = \prod_{i=1}^k \mu(G_i, x)$ .

**Lemma 2.2** [1] Let  $e = uv \in E(G)$ . Then  $\mu(G, x) = \mu(G - e, x) - \mu(G - \{u, v\}, x)$ .

**Lemma 2.3** [1] Let  $G$  be a connected graph and  $u \in V(G)$ ,  $e \in E(G)$ . Then  $M(G)$  is a single root of  $\mu(G, x)$  and  $M(G) > M(G - u)$ ,  $M(G) > M(G - e)$ .

**Lemma 2.4** [3] Let  $G$  be a connected graph. Then

(1)  $M(G) < 2$  if and only if  $G \in \Omega_1 = \{K_1, P_n, C_n, T(1, 1, n), T(1, 2, i) (2 \leq i \leq 4), D_{3,1}\}$ ;

(2)  $M(G) = 2$  if and only if  $G \in \Omega_2 = \{K_{1,4}, T(2, 2, 2), T(1, 3, 3), T(1, 2, 5), I_n, D_{3,2}, D_{4,1}\}$ .

**Lemma 2.5** [4] Let  $M(G) < 2$ . Then graph  $G$  is matching uniquely if and only if  $G = kK_1 \cup m_2P_2 \cup m_3P_3 \cup [\bigcup_{i \geq 2} m_{2i}P_{2i}] \cup [\bigcup_{j \geq 3} n_jC_j] \cup dD_{3,1} \cup eT(1, 2, 3) \cup fT(1, 2, 4)$ , where  $kn_j = m_i n_{i+1} = m_2d = m_3d = n_3e = n_{15}e = n_3n_5f = n_5n_9f = 0$  and  $k, m_i, n_j, d, e, f$  are non-negative integer.

**Lemma 2.6** [5] (1)  $P_{2m+1} \sqcap P_m \cup C_{m+1}$  ( $m \geq 2$ ) (2)  $T(1, 1, n) \sqcap K_1 \cup C_{n+2}$

(3)  $T(1, 2, 2) \sqcap P_2 \cup D_{3,1}$  (4)  $K_1 \cup C_6 \sqcap P_3 \cup D_{3,1}$

(5)  $K_1 \cup C_9 \sqcap C_3 \cup T(1, 2, 3)$  (6)  $K_1 \cup C_{15} \sqcap C_3 \cup C_5 \cup T(1, 2, 4)$

**Lemma 2.7** [6] (1)  $D_{3,2} \sqcap D_{4,1}$  (2)  $K_1 \cup D_{3,2} \sqcap I_6$

(3)  $T(2, 2, 2) \sqcap P_2 \cup D_{3,2}$  (4)  $T(1, 3, 3) \sqcap P_3 \cup D_{3,2}$

(5)  $T(1, 2, 5) \sqcap P_4 \cup D_{3,2}$  (6)  $K_1 \cup I_6 \sqcap P_2 \cup K_{1,4}$

(7)  $P_{m-4} \cup I_n \sqcap P_{n-4} \cup I_m$  ( $m, n \geq 6$ ) (8)  $I_{2m-3} \sqcap I_m \cup C_{m-3}$  ( $m \geq 6$ )

**Lemma 2.8** [5]  $G \sim H$  if and only if  $G^c \sim H^c$ .

### III. MAIN RESULTS

**Theorem 3.1** Let  $[P_2 \cup I_6 \cup T(1, 1, n)]$  be the classes of matching equivalent graphs of  $P_2 \cup I_6 \cup T(1, 1, n)$ .

Then

(1) If  $n \neq 1, 4, 7, 13$ ,  $[P_2 \cup I_6 \cup T(1, 1, n)] = \{2P_2 \cup K_{1,4} \cup C_{n+2}, K_1 \cup T(2, 2, 2) \cup T(1, 1, n), 2K_1 \cup T(2, 2, 2) \cup C_{n+2}, K_1 \cup P_2 \cup I_6 \cup C_{n+2}, K_1 \cup P_2 \cup D_{3,2} \cup T(1, 1, n), 2K_1 \cup P_2 \cup D_{3,2} \cup C_{n+2}, K_1 \cup P_2 \cup D_{4,1} \cup T(1, 1, n), 2K_1 \cup P_2 \cup D_{4,1} \cup C_{n+2}\}$ ;

(2) If  $n = 1$ ,  $[P_2 \cup I_6 \cup T(1, 1, 1)] = \{2P_2 \cup K_{1,4} \cup C_3, P_2 \cup K_{1,4} \cup P_5, K_1 \cup P_2 \cup I_6 \cup C_3, K_1 \cup I_6 \cup P_5, K_1 \cup P_2 \cup I_9, K_1 \cup P_2 \cup D_{3,2} \cup T(1, 1, 1), 2K_1 \cup P_2 \cup D_{3,2} \cup C_3, 2K_1 \cup P_5 \cup D_{3,2}, K_1 \cup P_2 \cup D_{4,1} \cup T(1, 1, 1), 2K_1 \cup P_2 \cup D_{4,1} \cup C_3, 2K_1 \cup P_5 \cup D_{4,1}\}$ ;

(3) If  $n = 4$ ,  $[P_2 \cup I_6 \cup T(1, 1, 4)] = \{K_1 \cup T(2, 2, 2) \cup T(1, 1, 4), 2K_1 \cup T(2, 2, 2) \cup C_6, K_1 \cup P_3 \cup D_{3,1} \cup T(2, 2, 2), K_1 \cup P_2 \cup C_6 \cup I_6, P_2 \cup P_3 \cup D_{3,1} \cup I_6, P_3 \cup T(1, 2, 2) \cup I_6, K_1 \cup P_2 \cup D_{3,2} \cup T(1, 1, 4), 2K_1 \cup P_2 \cup C_6 \cup D_{3,2}, K_1 \cup P_2 \cup P_3 \cup D_{3,1} \cup D_{3,2}, K_1 \cup P_3 \cup T(1, 2, 2) \cup D_{3,2}, K_1 \cup P_2 \cup D_{4,1} \cup T(1, 1, 4), 2K_1 \cup P_2 \cup C_6 \cup D_{4,1}, K_1 \cup P_2 \cup P_3 \cup D_{3,1} \cup D_{4,1}, K_1 \cup P_3 \cup T(1, 2, 2) \cup D_{4,1}\}$ ;

(4) If  $n = 7$ ,  $[P_2 \cup I_6 \cup T(1, 1, 7)] = \{K_1 \cup T(2, 2, 2) \cup T(1, 1, 7), 2K_1 \cup T(2, 2, 2) \cup C_9, K_1 \cup C_3 \cup T(1, 2, 3) \cup T(2, 2, 2), K_1 \cup P_2 \cup C_9 \cup I_6, P_2 \cup C_3 \cup T(1, 2, 3) \cup I_6, P_5 \cup T(1, 2, 3) \cup I_6, K_1 \cup P_2 \cup D_{3,2} \cup T(1, 1, 7), 2K_1 \cup P_2 \cup C_9 \cup D_{3,2}, K_1 \cup P_2 \cup C_3 \cup T(1, 2, 3) \cup D_{3,2}, K_1 \cup P_5 \cup$

$T(1,2,3) \cup D_{3,2}, K_1 \cup P_2 \cup D_{4,1} \cup T(1,1,7), 2K_1 \cup P_2 \cup C_9 \cup D_{4,1}, K_1 \cup P_2 \cup C_3 \cup T(1,2,3) \cup D_{4,1}, K_1 \cup P_5 \cup T(1,2,3) \cup D_{4,1}$ };

(5) If  $n=13$ ,  $[P_2 \cup I_6 \cup T(1,1,13)] = \{K_1 \cup T(2,2,2) \cup T(1,1,13), 2K_1 \cup T(2,2,2) \cup C_{15}, K_1 \cup C_3 \cup C_5 \cup T(1,2,4) \cup T(2,2,2), K_1 \cup P_2 \cup C_{15} \cup I_6, P_2 \cup C_3 \cup C_5 \cup T(1,2,4) \cup I_6, P_5 \cup C_5 \cup T(1,2,4) \cup I_6, K_1 \cup P_2 \cup D_{3,2} \cup T(1,1,13), 2K_1 \cup P_2 \cup C_{15} \cup D_{3,2}, K_1 \cup P_2 \cup C_3 \cup C_5 \cup T(1,2,4) \cup D_{3,2}, K_1 \cup P_5 \cup C_5 \cup T(1,2,4) \cup D_{3,2}, K_1 \cup P_2 \cup D_{4,1} \cup T(1,1,13), 2K_1 \cup P_2 \cup C_{15} \cup D_{4,1}, K_1 \cup P_2 \cup C_3 \cup C_5 \cup T(1,2,4) \cup D_{4,1}, K_1 \cup P_5 \cup C_5 \cup T(1,2,4) \cup D_{4,1}\}$ .

**Proof.** Let  $H \sim P_2 \cup I_6 \cup T(1,1,n)$ , then  $M(H) = M(P_2 \cup I_6 \cup T(1,1,n))$ , By Lemma 2.4, we obtain  $M(H) = M(I_6) = 2$ . There must be a connected component  $H_1$  in  $H$  belonging to the set  $\Omega_2$ . Let  $H = H_1 \cup H_2$ . Thus, we distinguish with the following cases according to  $M(I_6)$ .

**Case 1:** If  $H_1 = K_{1,4}$ , then  $P_2 \cup I_6 \cup T(1,1,n) \sqsubseteq K_{1,4} \cup H_2$ . By  $K_{1,4} \cup H_2 \sqsubseteq P_2 \cup I_6 \cup T(1,1,n) \sqsubseteq P_2 \cup I_6 \cup K_1 \cup C_{n+2} \sqsubseteq 2P_2 \cup K_{1,4} \cup C_{n+2}$ , we have  $H_2 \sqsubseteq 2P_2 \cup C_{n+2}$ .

**Subcase 1.1:** If  $n \neq 1$ , then  $H_2 \sqsubseteq 2P_2 \cup C_{n+2}$ ;

**Subcase 1.2:** If  $n=1$ , by Lemma 2.6, we have  $H_2 \sqsubseteq 2P_2 \cup C_3 \sqsubseteq P_2 \cup P_5$ .

**Case 2:** If  $H_1 = T(2,2,2)$ , then  $P_2 \cup I_6 \cup T(1,1,n) \sqsubseteq T(2,2,2) \cup H_2$ . By  $T(2,2,2) \cup H_2 \sqsubseteq P_2 \cup I_6 \cup T(1,1,n) \sqsubseteq P_2 \cup K_1 \cup D_{3,2} \cup T(1,1,n) \sqsubseteq K_1 \cup T(2,2,2) \cup T(1,1,n)$ , we have  $H_2 \sqsubseteq K_1 \cup T(1,1,n)$ .

**Subcase 2.1:** If  $n \neq 4, 7, 13$ , By Lemma 2.6, we have  $H_2 \sqsubseteq K_1 \cup T(1,1,n) \sqsubseteq 2K_1 \cup C_{n+2}$ .

**Subcase 2.2:** If  $n=4$ , by Lemma 2.6, we have  $H_2 \sqsubseteq K_1 \cup T(1,1,4) \sqsubseteq 2K_1 \cup C_6 \sqsubseteq K_1 \cup P_3 \cup D_{3,1}$ ;

**Subcase 2.3:** If  $n=7$ , by Lemma 2.6, we have  $H_2 \sqsubseteq K_1 \cup T(1,1,7) \sqsubseteq 2K_1 \cup C_9 \sqsubseteq K_1 \cup C_3 \cup T(1,2,3)$ ;

**Subcase 2.4:** If  $n=13$ , by Lemma 2.6, we have  $H_2 \sqsubseteq K_1 \cup T(1,1,13) \sqsubseteq 2K_1 \cup C_{15} \sqsubseteq$

$K_1 \cup C_3 \cup C_5 \cup T(1,2,4)$ ;

**Case 3:** If  $H_1 = T(1,3,3)$ , then  $P_2 \cup I_6 \cup T(1,1,n) \sqsubseteq T(1,3,3) \cup H_2$ . By  $K_1 \cup P_2 \cup I_6 \cup T(1,1,n) \sqsubseteq K_1 \cup T(1,3,3) \cup H_2 \sqsubseteq K_1 \cup P_3 \cup D_{3,2} \cup H_2 \sqsubseteq P_3 \cup I_6 \cup H_2$ , we have  $P_3 \cup H_2 \sqsubseteq K_1 \cup P_2 \cup T(1,1,n) \sqsubseteq 2K_1 \cup P_2 \cup C_{n+2}$ . By Lemma 2.5 and Lemma 2.6, this case does not exist.

**Case 4:** If  $H_1 = T(1,2,5)$ , then  $P_2 \cup I_6 \cup T(1,1,n) \sqsubseteq T(1,2,5) \cup H_2$ . By  $K_1 \cup P_2 \cup I_6 \cup T(1,1,n) \sqsubseteq K_1 \cup T(1,2,5) \cup H_2 \sqsubseteq K_1 \cup P_4 \cup D_{3,2} \cup H_2 \sqsubseteq P_4 \cup I_6 \cup H_2$ , we have  $P_4 \cup H_2 \sqsubseteq K_1 \cup P_2 \cup T(1,1,n) \sqsubseteq 2K_1 \cup P_2 \cup C_{n+2}$ . By Lemma 2.5 and Lemma 2.6, this case does not exist.

**Case 5:** If  $H_1 = I_m$ , then  $P_2 \cup I_6 \cup T(1,1,n) \sqsubseteq I_m \cup H_2$ .

**Subcase 5.1:** If  $n \neq 1, 4, 7, 13$ , By Lemma 2.6, we have  $I_m \cup H_2 \sqsubseteq P_2 \cup I_6 \cup T(1,1,n) \sqsubseteq P_2 \cup I_6 \cup K_1 \cup C_{n+2}$ . So, if  $m=6$ , we have  $H_2 \sqsubseteq P_2 \cup T(1,1,n) \sqsubseteq P_2 \cup K_1 \cup C_{n+2}$ ;

**Subcase 5.2:** If  $n=1$ , By Lemma 2.6 and Lemma 2.7, we have  $I_m \cup H_2 \sqsubseteq P_2 \cup I_6 \cup T(1,1,1) \sqsubseteq P_2 \cup I_6 \cup K_1 \cup C_3 \sqsubseteq I_6 \cup K_1 \cup P_5 \sqsubseteq P_2 \cup K_1 \cup I_9$ . So, if  $m=6$ , we have  $H_2 \sqsubseteq P_2 \cup T(1,1,1) \sqsubseteq P_2 \cup K_1 \cup C_3 \sqsubseteq K_1 \cup P_5$ ; if  $m=9$ ,  $H_2 \sqsubseteq P_2 \cup K_1$ ;

**Subcase 5.3:** If  $n=4$ , By Lemma 2.6, we have  $I_m \cup H_2 \sqsubseteq P_2 \cup I_6 \cup T(1,1,4) \sqsubseteq P_2 \cup I_6 \cup K_1 \cup C_6 \sqsubseteq P_2 \cup I_6 \cup P_3 \cup D_{3,1} \sqsubseteq I_6 \cup P_3 \cup T(1,2,2)$ . So, if  $m=6$ , we have  $H_2 \sqsubseteq P_2 \cup T(1,1,4) \sqsubseteq P_2 \cup K_1 \cup C_6 \sqsubseteq P_2 \cup P_3 \cup D_{3,1} \sqsubseteq P_3 \cup T(1,2,2)$ ;

**Subcase 5.4:** If  $n=7$ , By Lemma 2.6, we have  $I_m \cup H_2 \sqsubseteq P_2 \cup I_6 \cup T(1,1,7) \sqsubseteq P_2 \cup I_6 \cup K_1 \cup C_9 \sqsubseteq P_2 \cup I_6 \cup C_3 \cup T(1,2,3) \sqsubseteq I_6 \cup P_5 \cup T(1,2,3)$ . So, if  $m=6$ , we have  $H_2 \sqsubseteq P_2 \cup T(1,1,7) \sqsubseteq P_2 \cup K_1 \cup C_9 \sqsubseteq P_2 \cup C_3 \cup T(1,2,3) \sqsubseteq P_5 \cup T(1,2,3)$ ;

**Subcase 5.5:** If  $n=13$ , By Lemma 2.6, we have  $I_m \cup H_2 \sqsubseteq P_2 \cup I_6 \cup T(1,1,13) \sqsubseteq P_2 \cup I_6 \cup K_1 \cup C_{15} \sqsubseteq P_2 \cup I_6 \cup C_3 \cup C_5 \cup T(1,2,4) \sqsubseteq I_6 \cup P_5 \cup C_5 \cup T(1,2,4)$ . So, if  $m=6$ , we have  $H_2 \sqsubseteq P_2 \cup T(1,1,13) \sqsubseteq P_2 \cup K_1 \cup C_{15} \sqsubseteq P_2 \cup C_3 \cup C_5 \cup T(1,2,4) \sqsubseteq P_5 \cup C_5 \cup T(1,2,4)$ .

**Case 6:** If  $H_1 = D_{3,2}$ , then  $P_2 \cup I_6 \cup T(1,1,n) \sqsubseteq D_{3,2} \cup H_2$ . By Lemma 2.7, we have  $K_1 \cup P_2 \cup I_6 \cup T(1,1,n) \sqsubseteq K_1 \cup D_{3,2} \cup H_2 \sqsubseteq I_6 \cup H_2$ .

**Subcase 6.1:** If  $n \neq 1, 4, 7, 13$ , By Lemma 2.6, we have  $H_2 \sqsubseteq K_1 \cup P_2 \cup T(1,1,n) \sqsubseteq 2K_1 \cup P_2 \cup C_{n+2}$ ;

**Subcase 6.2:** If  $n=1$ , By Lemma 2.6, we have  $H_2 \sqsubseteq K_1 \cup P_2 \cup T(1,1,1) \sqsubseteq 2K_1 \cup P_2 \cup C_3 \sqsubseteq 2K_1 \cup P_5$ ;

**Subcase 6.3:** If  $n=4$ , By Lemma 2.6, we have  $H_2 \sqsubseteq K_1 \cup P_2 \cup T(1,1,4) \sqsubseteq 2K_1 \cup P_2 \cup C_6 \sqsubseteq K_1 \cup P_2 \cup P_3 \cup D_{3,1}$ ;

**Subcase 6.4:** If  $n=7$ , By Lemma 2.6, we have  $H_2 \sqsubseteq K_1 \cup P_2 \cup T(1,1,7) \sqsubseteq 2K_1 \cup P_2 \cup C_9 \sqsubseteq K_1 \cup P_2 \cup C_3 \cup T(1,2,3) \sqsubseteq K_1 \cup P_5 \cup T(1,2,3)$ ;

**Subcase 6.5:** If  $n=13$ , By Lemma 2.6, we have  $H_2 \sqsubseteq K_1 \cup P_2 \cup T(1,1,13) \sqsubseteq 2K_1 \cup P_2 \cup C_{15} \sqsubseteq K_1 \cup P_2 \cup C_3 \cup C_5 \cup T(1,2,4) \sqsubseteq K_1 \cup P_5 \cup C_5 \cup T(1,2,4)$ ;

**Case 7:** If  $H_1 = D_{4,1}$ . By  $D_{3,2} \sqsubseteq D_{4,1}$ , it is similar to Case 6. The proof is omitted.

The proof of Theorem 3.1 is complete.

By Theorem 3.1 and Lemma 2.8, we have Theorem 3.2:

**Theorem 3.2** Let  $[(P_2 \cup I_6 \cup T(1,1,n))^c]$  be The classes of matching equivalent graphs of  $(P_2 \cup I_6 \cup T(1,1,n))^c$ . Then

(1) If  $n \neq 1, 4, 7, 13$ ,  $[(P_2 \cup I_6 \cup T(1,1,n))^c] = \{(2P_2 \cup K_{1,4} \cup C_{n+2})^c, (K_1 \cup T(2,2,2) \cup T(1,1,n))^c, (2K_1 \cup T(2,2,2) \cup C_{n+2})^c, (K_1 \cup P_2 \cup I_6 \cup C_{n+2})^c, (K_1 \cup P_2 \cup D_{3,2} \cup T(1,1,n))^c, (2K_1 \cup P_2 \cup D_{3,2} \cup C_{n+2})^c, (K_1 \cup P_2 \cup D_{4,1} \cup T(1,1,n))^c, (2K_1 \cup P_2 \cup D_{4,1} \cup C_{n+2})^c\}$ ;

(2) If  $n=1$ ,  $[(P_2 \cup I_6 \cup T(1,1,1))^c] = \{(2P_2 \cup K_{1,4} \cup C_3)^c, (P_2 \cup K_{1,4} \cup P_5)^c, (K_1 \cup P_2 \cup I_6 \cup C_3)^c, (K_1 \cup I_6 \cup P_5)^c, (K_1 \cup P_2 \cup I_9)^c, (K_1 \cup P_2 \cup D_{3,2} \cup T(1,1,1))^c, (2K_1 \cup P_2 \cup D_{3,2} \cup C_3)^c, (2K_1 \cup P_5 \cup D_{3,2})^c, (K_1 \cup P_2 \cup D_{4,1} \cup T(1,1,1))^c, (2K_1 \cup P_2 \cup D_{4,1} \cup C_3)^c, (2K_1 \cup P_5 \cup D_{4,1})^c\}$ ;



(3) If  $n = 4$ ,  $[(P_2 \cup I_6 \cup T(1,1,4))^c] = \{(K_1 \cup T(2,2,2) \cup T(1,1,4))^c, (2K_1 \cup T(2,2,2) \cup C_6)^c, (K_1 \cup P_3 \cup D_{3,1} \cup T(2,2,2))^c, (K_1 \cup P_2 \cup C_6 \cup I_6)^c, (P_2 \cup P_3 \cup D_{3,1} \cup I_6)^c, (P_3 \cup T(1,2,2) \cup I_6)^c, (K_1 \cup P_2 \cup D_{3,2} \cup T(1,1,4))^c, (2K_1 \cup P_2 \cup C_6 \cup D_{3,2})^c, (K_1 \cup P_2 \cup P_3 \cup D_{3,1} \cup D_{3,2})^c, (K_1 \cup P_3 \cup T(1,2,2) \cup D_{3,2})^c, (K_1 \cup P_2 \cup D_{4,1} \cup T(1,1,4))^c, (2K_1 \cup P_2 \cup C_6 \cup D_{4,1})^c, (K_1 \cup P_2 \cup P_3 \cup D_{3,1} \cup D_{4,1})^c, (K_1 \cup P_3 \cup T(1,2,2) \cup D_{4,1})^c\}$ ;

(4) If  $n = 7$ ,  $[(P_2 \cup I_6 \cup T(1,1,7))^c] = \{(K_1 \cup T(2,2,2) \cup T(1,1,7))^c, (2K_1 \cup T(2,2,2) \cup C_9)^c, (K_1 \cup C_3 \cup T(1,2,3) \cup T(2,2,2))^c, (K_1 \cup P_2 \cup C_9 \cup I_6)^c, (P_2 \cup C_3 \cup T(1,2,3) \cup I_6)^c, (P_5 \cup T(1,2,3) \cup I_6)^c, (K_1 \cup P_2 \cup D_{3,2} \cup T(1,1,7))^c, (2K_1 \cup P_2 \cup C_9 \cup D_{3,2})^c, (K_1 \cup P_2 \cup C_3 \cup T(1,2,3) \cup D_{3,2})^c, (K_1 \cup P_5 \cup T(1,2,3) \cup D_{3,2})^c, (K_1 \cup P_2 \cup D_{4,1} \cup T(1,1,7))^c, (2K_1 \cup P_2 \cup C_9 \cup D_{4,1})^c, (K_1 \cup P_2 \cup C_3 \cup T(1,2,3) \cup D_{4,1})^c, (K_1 \cup P_5 \cup T(1,2,3) \cup D_{4,1})^c\}$ ;

(5) If  $n = 13$ ,  $[(P_2 \cup I_6 \cup T(1,1,13))^c] = \{(K_1 \cup T(2,2,2) \cup T(1,1,13))^c, (2K_1 \cup T(2,2,2) \cup C_{15})^c, (K_1 \cup C_3 \cup C_5 \cup T(1,2,4) \cup T(2,2,2))^c, (K_1 \cup P_2 \cup C_{15} \cup I_6)^c, (P_2 \cup C_3 \cup C_5 \cup T(1,2,4) \cup I_6)^c, (P_5 \cup C_5 \cup T(1,2,4) \cup I_6)^c, (K_1 \cup P_2 \cup D_{3,2} \cup T(1,1,13))^c, (2K_1 \cup P_2 \cup C_{15} \cup D_{3,2})^c, (K_1 \cup P_2 \cup C_3 \cup C_5 \cup T(1,2,4) \cup D_{3,2})^c, (K_1 \cup P_5 \cup C_5 \cup T(1,2,4) \cup D_{3,2})^c, (K_1 \cup P_2 \cup D_{4,1} \cup T(1,1,13))^c, (2K_1 \cup P_2 \cup C_{15} \cup D_{4,1})^c, (K_1 \cup P_2 \cup C_3 \cup C_5 \cup T(1,2,4) \cup D_{4,1})^c, (K_1 \cup P_5 \cup C_5 \cup T(1,2,4) \cup D_{4,1})^c\}$ .

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#### REFERENCES

- [1] Godsil C D, "Algebraic Combinatorics", New York : Chapman and Hall, 1993
- [2] Bondy J A, Murty U S R, "Graph Theory with Applications", North-Holland: Amsterdam, 1976
- [3] Haicheng Ma, "GRAPHS CHARACTERIZED BY THE ROOTS OF MATCHING", Journal of Qufu Normal University, vol. 27, no.1, pp: 33-36, 2001
- [4] Haicheng Ma, "The matching equivalent classes of graphs with the maximum matching root less than 2", Journal of Systems Science and Mathematical Sciences, vol. 23, no.3, pp: 337- 342, 2003
- [5] Haicheng Ma, "On the Matching Equivalent Classes of Two Kind of Graphs", Journal of Mathematical Study, vol.33,no.2, pp: 218-222, 2000
- [6] Haicheng Ma, "The Matching Equivalent Classes of Graphs of I Shape", Journal of Mathematical Study, vol.35,no.1, pp: 65-71, 2002



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