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Stress Analysis of Functionally Graded Beam and Finite Element Analysis using ANSYS Workbench

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ABSTRACT: The extensive use of Functionally Graded (FG) materials are found in aerospace, automobile, defence industries. Material tailoring of FG beam can be design to have desired distribution of stress in FG beam. The Elastic property of FG beam along the axis here is varied linearly, quadratic, exponentially with constant Poisson's ratio. The effect various gradation on deflection and stress in FG beam is studied. A quasi-static condition without body force is assume while doing static analysis of FG beam. An FG cantilever beam with free end subjected to point load and uniformly distributed load over the span length is simulated. Independent Finite Element modelling of functionally Graded beam is done by using ANSYS© workbench. Euler Bernoulli's theory is used for obtaining in-plane stress and displacement. These results are validated through finite element analysis in ANSYS© workbench. It shows that the grading of the beam along the axis plays vital role in reducing deflection in beam.

KEYWORDS: FG Beam, Finite Element Analysis, Modelling.

I. INTRODUCTION

Through their experimental work on single edge notch specimens made from ethylene carbon monoxide copolymer, Li et al. (1) showed that material gradation can serve as a means to enhance the fracture resistance response. One of the reason of failure of beam is stress concentration. One of the way to reduce the stress concentration in components subjected to traction loading is to use smooth grading of Mechanical properties of materials (2). The category of inhomogeneous material wherein the material properties are positional dependent in a specific manner is known as Functionally graded (FG) materials (3; 2). The FG beams will have Elastic properties varying from a point to point in one direction or multiple direction. The variation of Elastic properties of FG beams found in literature are power law, sigmoid and exponential models (4).

Reddy (5) worked on a homogeneous cantilever beam to do finite element analysis by using quadratic element with 8 nodes. Pydah and Sabale analysed the statics analysis of bi-directional power law FG circular beams under assumptions of the classical hairbrush hypothesis (6). Shear deformation theory with higher order was used by Kadoli et al. to study the static behavior of FG metal-ceramic beams. The transverse displacement, stresses under the action of uniform distributed load were found (7). Sankar studied a FGM beam subjected to sinusoidal transverse loading. Along the thickness of beam, FG beam with Exponential variation in the material property (Modulus of Elasticity) was assumed with constant Poisson ratio (8). Chawde and Bhandakkar analysed a mixed boundary value problem in FG cylinder subjected to variety of traction loading and mentioned method to model power law FG material (9). Khadke and chawde carried the finite element analysis of pressure vessel subjected to various loading like wind pressure and gravity effect by using ANSYS workbench (10).

The present work here gives a static analysis of the cantilever FG beam subjected two different loading cases as: (i) Free end subjected to point load and (ii) uniformly distributed load (UDL) over complete length of a FG beam. The Elastic property (Modulus of Elasticity) of the cantilever beam is assume to vary along the axis of the beam in linear, exponential, quadratic form with constant Poisson's ratio.

The problem formulation is described in next section succeeded by section explaining the solution methodology to formulate field solution for stress and displacement of FG beam subjected to transverse load followed by its application to example problems. Thereafter detailed discussion on results is given in the next section. Validation of the present work is done through independent ANSYS© based FE solution. The Conclusions section is encapsulated at the end.

II. PROBLEM DEFINITION AND FORMULATION

Consider a Functionally graded linear elastic beam having length L and cross-section as standard I section as shown in Figure 2. The modulus of Elasticity of the beam assumed to linearly varying along the normal to cross-section of the beam as (Fig. 1):

$$E(x) = E_0(1 + x/L) \tag{1}$$

Where E_0 represent modulus of Elasticity at fully constrained end of the beam as shown in Figure 2. The quadratic variation of Modulus of Elasticity for FG beam is given as (4):

$$E(x) = (E_L - E_R)(1 - x/L)^k + E_R \tag{2}$$

Where $k = 2, E_L = 200\text{GPa}, E_R = 400\text{GPa}$.

The exponential form for Modulus of Elasticity for FG Beams is given as:

$$E(x) = (E_o)e^{(x/L)\text{LN}(2)} \tag{3}$$

Where $E_0 = 200\text{GPa}$ in our calculation.

The normal stress and displacement components of beam are denoted by $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ and u_{xx}, u_{yy}, u_{zz} respectively. Modulus of Elasticity of beam is varying linearly as mention in Eq. 1 along axis and Poisson's ratio ν is assumed to be constant through all direction of the FG beam.

Two type of loading scenario are considered here as: (a) A cantilever FG beam with free end subjected to constant point load (b) A cantilever FG beam with UDL w acting for whole span length L .

The planar equilibrium equations in $(x - y)$ with quasi-static conditions in index notation

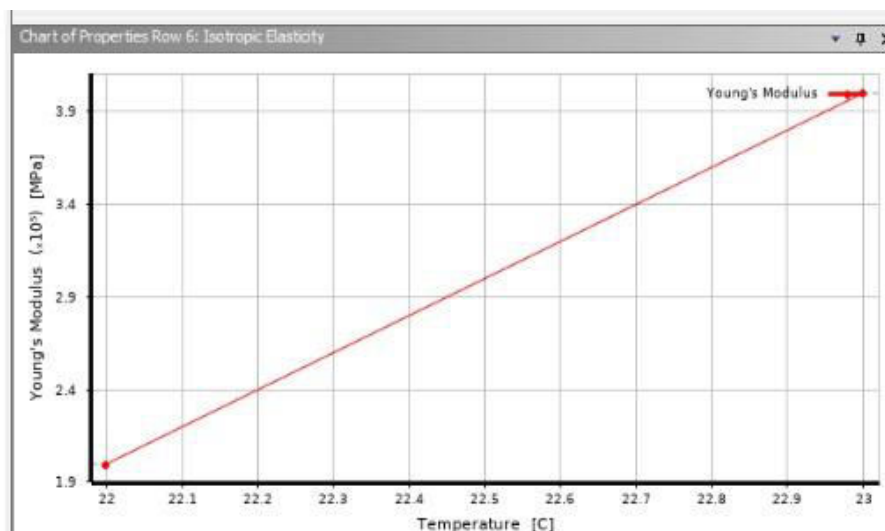


Figure 1: Modulus of Elasticity variation (Eq. (1)) with Temperature along the length of the FG beam in axial direction with constant Poisson ratio.



where index i take three values - 1,2,3 corresponds to three equations, are given as (11),

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0 \tag{4}$$

The compatibility equation in term of stress is given as

$$\frac{\partial \sigma_{xx}}{\partial y^2} - \nu \frac{\partial^2 \sigma_{yy}}{\partial y^2} - 2(1 + \nu) \frac{\partial^2 \sigma_{xy}}{\partial x \partial y} + \frac{\partial^2 \sigma_{yy}}{\partial x^2} - \nu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = 0 \tag{5}$$

The strain-displacement relations in the plane are given as (11),

$$\epsilon_{xx} = \frac{\partial u_x}{\partial x}, \epsilon_{yy} = \frac{\partial u_y}{\partial y}, \epsilon_{zz} = \frac{\partial u_z}{\partial z} \tag{6}$$

The generalized Hooke's law in planar form which relate stress with strain components is given as (11),

$$\epsilon_{xx} = \frac{1}{8} \frac{\sigma_{xx}(1 + \kappa) - (3 - \kappa)\sigma_{yy}}{\mu(x)}, \epsilon_{yy} = \frac{1}{8} \frac{\sigma_{yy}(1 + \kappa) - (3 - \kappa)\sigma_{xx}}{\mu(x)}, \epsilon_{xy} = \frac{1}{2} \frac{\sigma_{xy}}{\mu(x)} \tag{7}$$

where κ is the Kolosov constant and take the values equal as:

$$\begin{aligned} \kappa &= (3 - \nu)/(1 + \nu) \\ \kappa &= (3 - 4\nu) \end{aligned} \tag{8}$$

Eq. 8a for plane stress ($P\sigma$), Eq. 8b for plane strain ($P\epsilon$) deformation and $\mu(x)$ is shear Modulus of the beam. The BCs must be specified along with Eqs. (4, 6), to complete problem of elasticity in a complete sense. (11; 12). The effect of transverse load P and w is to bend the beam and deflect. The next section outlines the detailed procedure to find field solution of stress and displacement for FG beam subjected to transverse load P and w .

III. METHODOLOGY FOR STRESS-DISPLACEMENT FIELD IN FG BEAM

Consider a FG cantilever beam as shown in Figure 2a,b subjected to loads as P and $udlw$ respectively. The stress and deflection field in the beam due to transverse loading can be found using Euler Bernoulli's theory of bending. The immediate effect of transverse load P to produce bending stress and is given as (11; 12):

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R} \tag{9}$$

Where M is bending moment on beam, I is Area Moment of Inertia (M.I) of cross-section of beam about bending axis which is xx axis here, σ_b is bending stress at a layer y from bending axis xx. E represent the modulus of Elasticity for beam materials and R is beam curvature radius. Using Eq. 9 the bending stress σ_b at layer y from bending xx is given as:

$$\sigma_b = \frac{M}{I} y \tag{10}$$

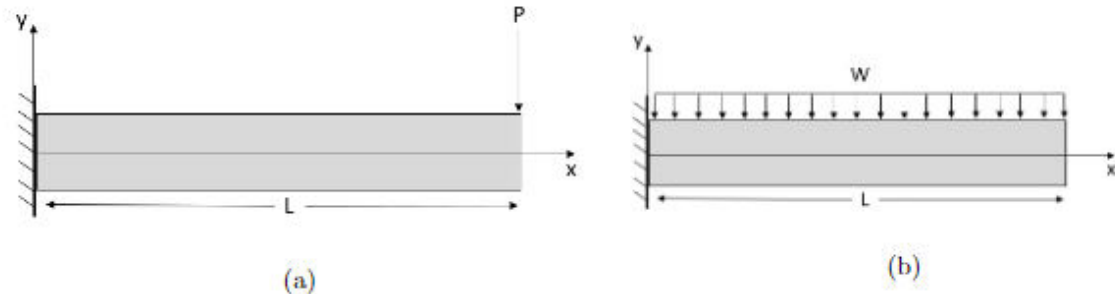


Figure 2: (a) FG beam having span length L with linearly varying Modulus of Elasticity (Eq. (1)) along the axis of FG beam and constant Poisson's ratio ν subjected to load P as shown, (b) FG beam having span length L with linearly varying Modulus of Elasticity (Eq. (1)) along the axis of the FG beam and constant Poisson's ratio ν subjected to udl w kN/m along complete span length L

The equation of the elastic curve for beam with variation of Modulus of Elasticity $E(x)$ (Eq. 1) along the axis, subjected to transverse load $p(x)$ is given as:

$$E(x)I \frac{\partial^2 y}{\partial x^2} = M \tag{11}$$

Above Eq. 11 can be solve to find the slope ($\frac{\partial y}{\partial x}$) and deflection (y) at any point along the span length L . The various methods to solve problems of beam in the literature are Rayleigh Method, Double integration Method, Airy Stress Function Method. In this paper we have used double integration method to find deflection of the FG beam when subjected to transverse load. Next section gives the detailed procedure for two example problems.

IV. EXAMPLE PROBLEMS

(i) A Cantilever FG beam having symmetrical I cross-section and span length l with free end subjected to constant point load P as shown in Fig. 2a:

Figure 2a shows a free end subjected to point load P in FG beam with linearly varying Modulus of Elasticity E as per Eq. 1 having I cross-section with span length L .

The displacement BCs along the fully constrained end of the cantilever FG beam shown in Fig. 1 at $x = 0$ is,

$$\begin{aligned} U_{x1} = U_{y1} = U_{z1} &= 0 \\ \gamma_{x1} = \gamma_{y1} = \gamma_{z1} &= 0 \\ \frac{\partial U}{\partial x} &= 0 \end{aligned} \tag{12}$$

Where U_{x1}, U_{y1}, U_{z1} represent linear displacement in x, y, z directional respectively and $\gamma_{x1}, \gamma_{y1}, \gamma_{z1}$ represent rotational displacement about x, y, z axis at fully constrained of the cantilever FG beam shown in Fig 2a.

The displacement equation FG cantilever beam in the first case as shown in Fig 2a is found by using double integration to Eq. 11 and subsequent BC's (Eq. 12) is given as:

$$U_{y1} = -\frac{PL^3}{IE_0} \left[2 \left(\frac{x}{L} + 1 \right) \text{LN} \left[\frac{x}{L} + 1 \right] - 0.5 \left(\frac{x}{L} \right)^2 - 2 \left(\frac{x}{L} \right) \right] \tag{13}$$



The maximum displacement will be at the free end and is given as

$$(U_{y1})_{MAX} = -\frac{PL^3}{IE_0} (4LN(2) - 2.5) \tag{14}$$

The values of the bracket on right side of the Eq. 14 is not found purposefully to keep the accuracy of the formula given for maximum displacement for a cantilever FG beam subjected to point load at free end shown in Fig. 2a. The calculation are done by taking, P = 10 kN, L = 2m, E0 = 200 GPa, I = 55208333.33 mm⁴. Figure 3 shows the bending stress and transverse deflection in the FG beam subjected to point load at free end as shown in Fig. 2a. The displacement equations for other cases where Modulus of Elasticity is varying in quadratic, Exponential form are not given here.

(ii) A Cantilever FG beam having symmetrical I cross-section and span length L subjected to UDL w kN/m as shown in Fig. 2b:

Figure 2b shows a FG beam with linearly varying Modulus E(x) as per Eq. 1 having I cross-section with span length L subjected to UDL as shown in Fig. 2b. The Modulus

E(x) is varied Linearly as Eq. (1)

The displacement and slop BCs along the fully constrained end of the cantilever FG beam at x = 0 is,

$$\begin{aligned} U_{x2} = U_{y2} = U_{z2} &= 0 \\ \gamma_{x2} = \gamma_{y2} = \gamma_{z2} &= 0 \\ \frac{\partial U}{\partial x} &= 0 \end{aligned} \tag{15}$$

Where U_{x2}, U_{y2}, U_{z2} represent linear displacement in x,y,z directional respectively and $\gamma_{x2}, \gamma_{y2}, \gamma_{z2}$ represent rotational displacement about x,y,z axis at fixed end of the cantilever FG beam shown in Fig. 2a.

The displacement equation for case ii where UDL is acting on FG cantilever beam through the span length l as shown in Fig. 2b was found by double integration to Eq. 11 and using BC's given in Eq. 15. The deflection is given as:

$$U_{y2} = -\frac{wL^4}{2IE_0} \left[\frac{1}{6} \left(\frac{x}{L}\right)^3 - 3 \left(\frac{x}{L}\right)^2 + 4 \left(\frac{x}{L} + 1\right) LN \left(\frac{x}{L} + 1\right) \right] \tag{16}$$

The maximum displacement will be at the free end and is given as

$$(U_{y2})_{MAX} = -\frac{wL^4}{2IE_0} \left[8LN(2) - \frac{16}{3} \right] \tag{17}$$

The calculation are done by taking parameters as: w = 200 kN, L = 2m, E0 = 200 GPa, I = 55208333.33 mm⁴. The next section gives the modelling method for FG beam and Finite Element analysis of FG Beam in ANSYS workbench.

V. MODELLING FG BEAM AND FINITE ELEMENT ANALYSIS

One of the well-known methods to model FG beam is to give position dependant Mechanical Properties (13; 9). The linearly (Eq. 1) varying Modulus of Elasticity E along the axis for cantilever beam was given temperature dependant as shown in Fig. 1 while Poisson’s ratio was given constant. To nullify the effect of temperature, coefficient of thermal Expansion was given as zero. On similar vein the Modulus of Elasticity was varied exponentially (Eq. 3) and Quadratic

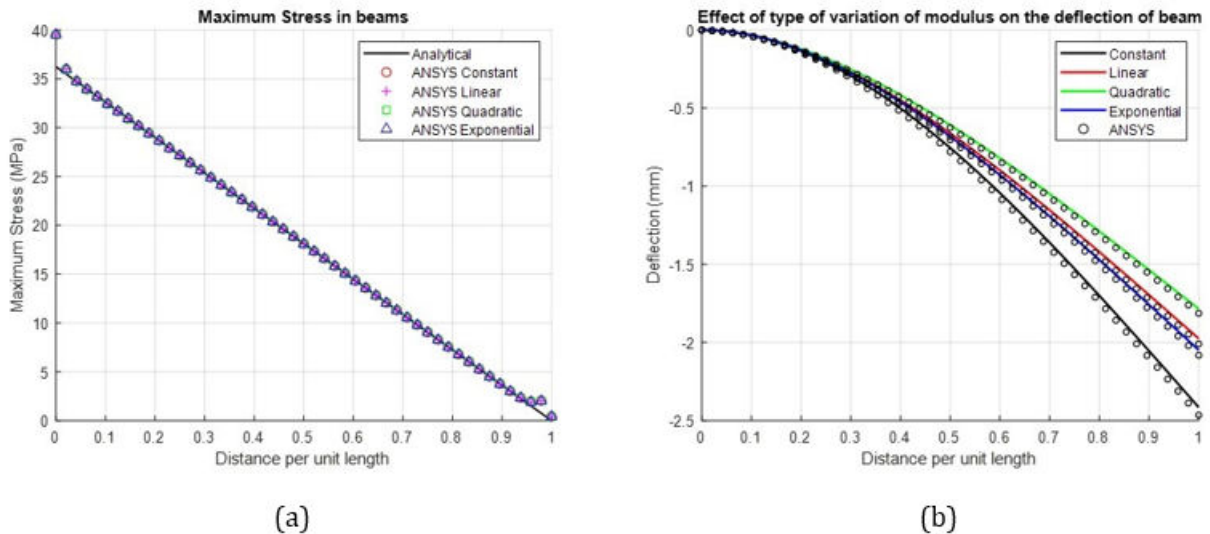


Figure 3: (a) Variation of bending stress along the length of the beam on top fiber of FG beam with constant, linear, quadratic, exponential Modulus of Elasticity, subjected to point load (Fig. 2a) at free end, (b) Variation of transverse Deflection of FG beam with constant, linear, quadratic, exponential Modulus of Elasticity when subjected to point load (Fig. 2a) at the free end of beam as acquired from the present work (continuous line) and ANSYS based FE solution (circular marks) for a constant, Linear, Quadratic, Exponential Modulus of Elasticity.

(Eq. 2) along the length of the beam. The structured mesh is used for full cantilever FE model. Total number of four nodedquadra elements used in FE model of FG beam in ANSYS is 7370, number of Nodes equal to 37587 with finer mesh (14; 15).

VI. RESULTS AND DISCUSSIONS

To start with Fig. 3a shows the variation of bending stress on top fiber along the length of the beam. The bending stress as expected is reducing along the length of the beam. It is maximum at the free end on the top fiber and reduces over the length. Fig. 3b shows the transverse deflection of the FG beam with I cross-section when subjected to transverse load. The Modulus of Elasticity of the beam was varied along the length of the beam and Poisson ratio was assumed to be constant. The variation of Modulus of Elasticity was assumed to be constant, linear, quadratic and exponential along the axis of a beam in both problems solved here. As expected, the bending stress distribution was linear in nature and is in

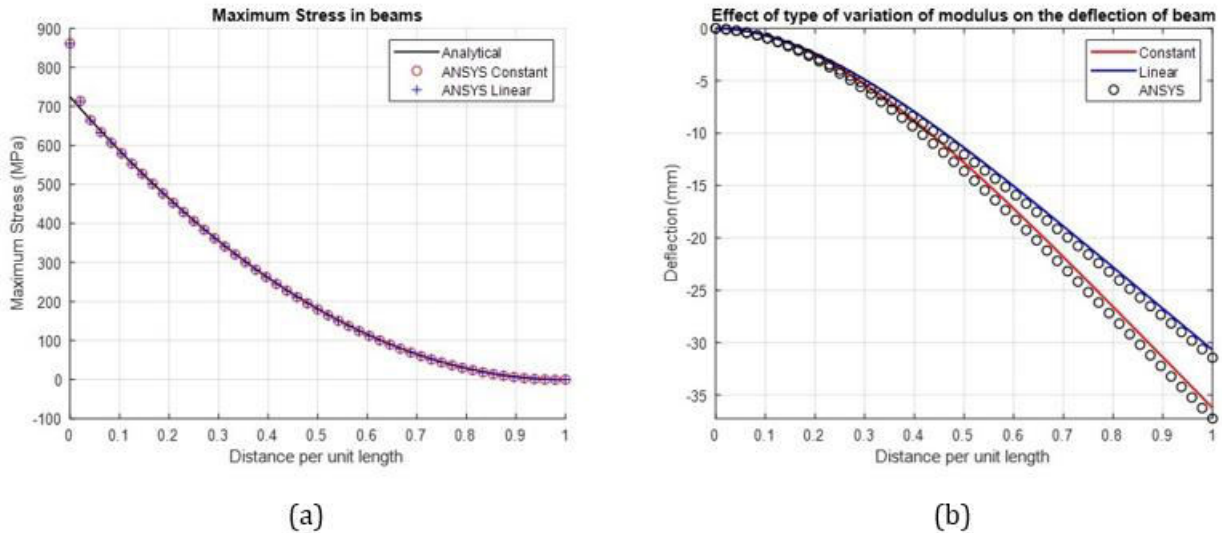


Figure 4: (a) Variation of bending stress along the length of the beam on top fiber of FG beam with constant and linear Modulus of Elasticity, subjected to UDL (Fig. 2b) over span length l , (b) Variation of transverse deflection of FG beam with constant, linear, Modulus of Elasticity when subjected to UDL (Fig. 2b) over span length l as acquired from the present work (continuous line) and ANSYS based FE solution (circular marks) for a constant, Linear Modulus of Elasticity

agreement with results in literature. Displacement solution was found for each case. Fig. 3 shows that there is flattening of displacement gradient due to grading of beam along the axis. There is reduction in maximum transverse deflection of FG beam also as compared to constant Modulus of Elasticity case. Figure 4a shows the bending stress distribution for the FG cantilever beam subjected to uniformly distributed over a span length. The transverse deflection in case of FG beam subjected to uniformly distributed is shown in Figure 4b. It shows that the excellent agreement with analytical solution found using Euler Bernoulli beam theory. The present work is also validated through excellent comparison with Finite element simulation done through ANSYS shown through open circles in Fig. 3, 4.

VI. CONCLUSION

The Euler Bernoulli Theory is used to find the transverse deflection of cantilever FG beam subjected to point load and UDL over span length. Functional grading of Modulus of Elasticity is assumed to be Linear, Exponential, quadratic long the axial direction of FG Beam with constant Poisson's ratio. In the case where FG cantilever beam is subjected to UDL over a span length, the Modulus of Elasticity is varied in Linearly. A quasi-static condition without body force is assume while doing static analysis of FG beam. Results shows that grading of FG beam contributes to reduction in transverse displacement. There is flattening of displacement gradient due to grading of beam along the axis. Thus, the flattening of gradient of field variables achieved through a functionally graded system can be further enhanced through material tailoring.

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