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## Ensuring the Accuracy and Transparency of the Mamdani Fuzzy Model when Training Experimental Data

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**ABSTRACT:** The purpose of this article is to study a new way to preserve the transparency of the fuzzy Mamdani model and improve its accuracy when training experimental data. First, the Mamdani fuzzy model is described, model training is formalized in the form of the problem of nonlinear optimization, and the ways to improve the accuracy of training are analyzed. Then the transparency requirements of Mamdani fuzzy model are formulated and the main methods of its preservation are analyzed.

KEYWORDS: Mamdani model, improve, function, coefficients of the membership function, fuzzy rule

In this article, we examine models where the relationship between inputs and outputs is described by a knowledge base of fuzzy "If — then" rules. In fuzzy modeling, a knowledge base in the Mamdani format is often used, where the antecedents and consequents of rules are set by fuzzy sets, such as "Low", "Average", "High", and etc. Fuzzy rules in this format are proposed in [1], on the basis of which E. Mamdani and S. Assilian developed the first fuzzy controller [2]. Unlike "black box" models, Mamdani's fuzzy models are transparent, and their structures are meaningfully interpreted in terms that are understandable not only to developers with high mathematical qualifications, but also to customers — doctors, economists, and managers.

To improve the accuracy of this model, it is proposed to configure additional parameters — the boundaries of the fuzzy set carrier in the consecutions of rules; and to preserve its transparency, the number of managed variables is reduced and new restrictions are introduced. The typical and proposed training schemes are compared in the example of predicting the fuel efficiency of a car.

Let us write Mamdani's fuzzy knowledge base as follows [1, 2, 4, 5]:

if 
$$(x_1 = \tilde{\alpha}_{1j} \text{ and } x_2 = \tilde{\alpha}_{2j} \text{ and ... and } x_n = \tilde{\alpha}_{nj} \text{ weighing } w_j$$
, then  $y = \tilde{d}_j$ ,  $j = 1, m_{(1)}$ , where  $\tilde{\alpha}_{ij}$  is fuzzy

term evaluating the variable  $x_i$  in the j-th rule, t = 1, n;

 $\tilde{d}_j$  is fuzzy inference of the j<sup>th</sup> rule; m is the number of rules in knowledge base;

 $w_j \in [0,1]$  - the weight coefficient reflecting the adequacy of the j<sup>th</sup> rule.

The Mamdani knowledge base can be interpreted as dividing the space of influencing factors into zones with blurred borders, within which the response function takes a fuzzy value. The number of such fuzzy zones is equal to the number of rules.

We will introduce the following notation that are necessary for further presentation of the material:

$$\mu_{j}, (x_{i}) - \text{the function of the input} \begin{array}{c} x_{i} \in \left[\underline{x_{i}}, x_{i}\right] \text{ belonging to the fuzzy term } \widetilde{\alpha}_{ij} \text{ i.e.} \\ \widetilde{\alpha}_{ij} = \int_{x_{i} \in \left[\underline{x_{i}}, \overline{x_{i}}\right]} (x_{i}) / x_{i} \mu_{j} \text{ ;} \\ y \in \left[\underline{y}, \overline{y}\right] \text{ } \widetilde{d} \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) / y \text{ } d \quad \widetilde{d}_{i} = \int_{z_{i} \in \left[\underline{y}, \overline{y}\right]} (y) /$$

 $\mu_{d_j}(y) = \text{the function of the output } y \in \left[\underline{y}, \overline{y}\right] \text{ belonging to the fuzzy term } \widetilde{d}_j, \text{ i.e. } \frac{d_j}{d_j} = \int_{y \in (\underline{y}, \overline{y})} (y) / y \mu_{d_j}$ 

The degree of fulfillment of the j-th rule premise for the current input vector  $X^* = (x_1^*, x_2^*, \dots, x_n^*)$  is calculated as follows:



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$$\mu_{j}(X^{*}) = w_{j} \cdot (\mu_{j}(x_{1}^{*}) \land (\mu_{j}(x_{2}^{*}) \land \dots \land (\mu_{j}(x_{n}^{*})), (1))$$
  
$$j = \overline{1, m},$$

where  $\wedge$  is the t-norm which in Mamdani fuzzy inference is usually implemented by the minimum operation. As a result of logical inference by the j-th rule of the knowledge base, we get such a fuzzy value of the output vari-

As a result of logical inference by the j-th rule of the knowledge base, we get such a fuzzy value of the output variable y:

$$\tilde{d}_{j}^{*} = imp(\tilde{d}_{j}, \mu_{j}(X^{*})), \ j = \overline{1, m},$$
(2)

where imp is an implication that in Mamdani fuzzy inference is implemented by the minimum operation, i.e. by "slicing" the membership function  $\mu_{d_j}(y)$  by level  $\mu_j(X^*)$ . Mathematically it is written in this way:

$$\widetilde{d}_{j}^{*} = \int_{y \in [\underline{y}, \overline{y}]} \min \left( \mu_{j}(X^{*}), \mu_{d_{j}}(y) \right) / y.$$

The resulting fuzzy set  $\tilde{y}^*$  is obtained by combining fuzzy sets (2):

$$\widetilde{y}^* = \widetilde{d}_1^* \bigcup \widetilde{d}_2^* \bigcup \ldots \bigcup \widetilde{d}_m^*,$$

which corresponds to the maximum operation on membership functions:

$$\mu_{y} \cdot (y) = \max (\mu_{d_{1}} \cdot (y), (\mu_{d_{2}} \cdot (y), ..., (\mu_{d_{m}} \cdot (y))).$$

The crisp value of the output  $\overset{y}{\widetilde{y}}$  corresponding to the input vector  $\overset{X}{\widetilde{y}}$  is determined by defuzzification of the  $\widetilde{\widetilde{y}}^*$ 

fuzzy set ´

The most commonly used defuzzification method is the center of gravity:

$$y^* = \int_{\underline{y}}^{\overline{y}} y\mu_y * (y) dy / \int_{\underline{y}}^{\overline{y}} \mu_y * (y) dy,$$

which provides the best dynamics of fuzzy model training Let the training sample that links the inputs to the output of the studied dependency be denoted by:

$$X_r, y_r, r = 1, M,$$
 (3)

where  $X_r = (x_{r1}, x_{r2}, ..., x_{rn})$  is input vector in the r-th data pair,  $y_r$  is corresponding output, M is sample size. For the mathematical formulation of the problem of training a fuzzy model from a sample (3), we introduce the following notation:

- P vector of parameters of membership functions terms of input and output variables;
- W vector of weight coefficients of knowledge base rules;
- $F(P, W, X_r)$  output result from the Mamdani fuzzy knowledge base with parameters (P, W) at the value of inputs  $X_r$ . Fuzzy inference is based on the formulas from the previous section of the article.
  - According to [4-8], training a fuzzy model consists in finding such a vector (P, W) that



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$$RMSE(P, W) = \sqrt{\frac{1}{M}} \sum_{r=\overline{1,M}} (y_r - F(P, W, X_r))^2 \to \min.$$
(4)

When configuring membership functions, there are five ways in which the coordinates of the vector P are set by:

1) coefficients of the parametric membership function for each fuzzy term [4-8]. For example, the triangular membership function is defined by three coefficients that correspond to the core and boundaries of the fuzzy set carrier.

$$|P| = \sum_{u} k_u$$

The length of the vector P is calculated as follows:  $\mu = \overline{1, N}$ , where  $k_u$  is the number of coefficients of the membership function of the u-th fuzzy term; N is the number of fuzzy terms in the knowledge base (1);

$$|P| = 2 \sum_{u \in \overline{U}} h_u$$

 $\mu = 1, N$ , where  $h_u$  is the number of boundaries of the  $\alpha$ -sections of each fuzzy term [7]. In this case 2)  $\alpha$  -sections of the u-th fuzzy term. For example, to set the  $\alpha$ -sections at levels 0, 0.5, and 1, six coefficients are needed - (left<sub>0</sub>, right<sub>0</sub>, left<sub>0,5</sub>, right<sub>0,5</sub>, left<sub>1</sub>, right<sub>1</sub>). (Fig. 1);

linguistic quantifiers such as "very", "more or less", etc. [9, 10]. In this case, |P| = N. The quantifier "very" 3) concentrates the fuzzy set, and the quantifier "more or less" smears it. The operations of concentration and smearing of fuzzy sets are implemented by raising the membership function to the power of 2 and 1/2, respectively [1];

compression-stretching coefficients of fuzzy terms [9]. These coefficients correspond to indicators of the de-4) gree to which the membership function is raised. The effect of this coefficient on the graph of the membership function is illustrated in Fig. 2. The values of 2 and 1/2 of the compression-tension coefficient are equivalent to the quantifiers "very" and "more-less»;

coefficients of the membership function, linguistic quantifier, and compression-extension coefficient for each 5) fuzzy term [9]. This hybrid approach combines methods 1, 2, and 4. In this case, the dimension of the optimization problem (4) increases greatly, because the number of configurable coefficients of each fuzzy term increases by 2 in comparison with the first method.



Fig. 1. Compression-stretching coefficients of fuzzy terms



Fig. 2

Comparative analysis of how to configure the membership functions is summarized in table 1. The first method of configuring membership functions is most often used because of the good balance between accuracy and duration of training for the fuzzy model. This method will be used in this article.



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Table 1. Comparative analysis of how to configure the membership functions is summarized

Method	Advantages	Disadvantages
1	Differentiable target function	The type of membership function cannot be changed during configuration
2	Differentiable target function. An analytical model of membership functions is not required. High training accuracy is potentially achieva- ble	Large dimension of the optimization problem (4). There are many restrictions on managed varia- bles necessary to ensure convexity of fuzzy sets
3	Minimum dimension of the optimization prob- lem (4)	A discrete optimization problem with a small number of alternatives, which does not allow to fine-tune the fuzzy model
4	Minimum dimension of the optimization prob- lem (4) Differentiable target function	The cores of fuzzy sets are not configured.
5	High training accuracy is potentially achieva- ble	Mixed discrete-continuous optimization prob- lem of large dimension

#### 1. The transparency of the fuzzy model

We will consider Mamdani's fuzzy model transparent if these conditions are met:

1) the knowledge base is not contradictory or redundant, i.e. it does not contain rules with the same antecedents;

2) the knowledge base is consistent with the number of terms, i.e. each term appears in at least one fuzzy rule;

3) for an arbitrary input vector, the output is not an empty fuzzy set;

4) separately, each membership function is meaningfully interpreted, i.e. the corresponding fuzzy set is normal and convex [11, 12];

5) each term-set is meaningfully interpreted as follows:

• the number of terms is not too large for the expert to match each fuzzy set with a linguistic score [11-13]. Following the works [7, 14], it is advisable to limit the power of the term set from above to the "magic" number  $7 \pm 2$  [15];

• fuzzy sets of different terms should not be equivalent or nearly equivalent [11-13]. Therefore, the graphs of the membership functions of neighboring terms, such as "Low" and "Lower than average", must differ visually;

• the linear ordering of fuzzy sets must not be violated, i.e. for a term set {"low", "lower than average", ..., " high»} of the x variable:

$$\forall x : \mu_{low}(x) \le \mu_{lower than average}(x) \le \ldots \le \mu_{hight}(x).$$
(5)

In addition, it is desirable that the knowledge base is compact, i.e. it contains the minimum (or close to it) number of rules necessary for adequate modeling of the studied dependence. With a large number of input variables, the compactness of the knowledge base provides a hierarchical representation of the rules [7, 14].

After configuring the accessory functions, the following typical violations of the transparency of fuzzy models occur (Fig. 3):

a) strong similarity of membership functions of neighboring fuzzy sets ("Low" and "Lower than average" in Fig. 3), which may introduce contradictions in the knowledge base;

b) the loss of linear order term-sets using different smearing of membership functions on the interval [65, 82] fuzzy set "Average" is more than fuzzy set "Higher than average", and on the interval [0, 3] fuzzy set "Lower than average" is more than fuzzy set "Low", although it should be the opposite;

c) non-interpretability of the extreme term – reducing the values of the variable x from 8 to 0 reduces the degree of membership to the fuzzy term "Low", although it should be the opposite.

d) incomplete covering of the range of possible values of input variables by fuzzy sets — the numbers from the range [82, 88] do not belong to fuzzy sets.



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Fig. 3 The transparency of fuzzy models occur

#### 2. How to protect the transparency of a fuzzy model

The first two transparency conditions of the fuzzy model concern the knowledge base. Let us assume that they are satisfied with its formation and the rules do not change while training.

The third condition can be met by using fuzzy terms whose carrier is not narrower than the range of change of the corresponding input variable. The easiest way to do this is to use parametric membership functions with a domain of definition  $(-\infty;\infty)$ , such as a Gaussian curve:

$$\mu(x) = \exp\left(-(x-b)^2/2c^2\right),$$
 (6)

where b is coordinate of the maximum, c > 0 is the coefficient of concentration.

The fourth condition is easy to fulfill using parametric membership functions that define convex and normal fuzzy sets. In this case, you need to restrict the cores of fuzzy sets to the range of changes in the corresponding variable. For

example, for the membership function (6), this restriction is written as:  $b \in [\underline{x}, x]$ To meet the fifth condition, there are three approaches that use: 1) formal transparency criteria; 2) linguistic quantifiers for modifying membership functions; 3) semantic restrictions.

Using the first approach, we synthesize a certain transparency criterion T(P) of the fuzzy model and proceed from (4) to the following multi-criteria optimization problem [16]: find a vector (P,W) so that:

$$\begin{cases} RMSE(P,W) \to \min, \\ T(P) \to \max. \end{cases}$$
(7)

Penalty functions are used to solve problem (7) applying typical optimization methods [12]. It is also possible to convert one of the criteria to a constraint, which converts (7) to a conditional optimization problem.

The transparency of a fuzzy model is usually calculated basing on the following similarity coefficient of fuzzy sets [12, 13, 16]:

$$S(\widetilde{A}, \widetilde{B}) = \frac{\left|\sup p(\widetilde{A} \cap \widetilde{B})\right|}{\left|\sup p(\widetilde{A} \cup \widetilde{B})\right|},$$
(8)

where  $\widetilde{A}$  and  $\widetilde{B}$  are fuzzy sets for which the similarity coefficient is calculated; supp is the carrier of the fuzzy set;

is the power of the set.

The value of similarity coefficient is 1 if the fuzzy sets are equivalent, and 0 if the fuzzy sets do not intersect. Formula (8) is suitable for calculating the similarity coefficient of fuzzy sets with compact carriers.

If the carriers of fuzzy sets are  $(-\infty;\infty)$ , then formula (8) does not work. In this case, it can be modified by replacing the carriers with  $\alpha$ -sections:



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$$S(\widetilde{A}, \widetilde{B}) = \frac{\left| (\widetilde{A} \cap \widetilde{B})_{\alpha} \right|}{\left| (\widetilde{A} \cup \widetilde{B})_{\alpha} \right|},$$

where  $\alpha \ll 1$ , for example a=0.01

As a transparency indicator, one chooses the value that is the inverse of the average or maximum similarity coefficient for all pairs of fuzzy sets from the knowledge base. Using the coefficient (8) only protects against one violation of the transparency of the fuzzy model — the similarity of the membership functions of fuzzy terms, so its isolated use does not preserve the transparency of the fuzzy model.

According to the second approach, membership functions are configured by changing the linguistic quantifiers of fuzzy sets [9, 10]. In this case, the original membership functions are selected in such a way that any combination of quantifiers does not violate the transparency of term sets. The disadvantage of this approach is the low accuracy of fuzzy model settings.

For the third approach, the following restrictions on values of managed variables are introduced in (4):

fuzzy sets completely cover the range of possible values of input variables [11], i.e. any number from this range belongs to at least one fuzzy set with a non-zero degree; (9)

coordinates of maxima of membership functions for fuzzy terms are limited by the ranges of changes in the corresponding variables [5, 7, 11]; (10)

only the membership functions of neighboring fuzzy terms intersect [11]; (11)

the intersection height of fuzzy sets of neighboring terms is bounded from below and above [17]; (12)

the distance between coordinates of maxima of functions and neighboring terms is bounded from below [5, 13]; (13)

the length of a certain  $\alpha$ -section of each fuzzy set is bounded from below and from above [16]. (14)

Restrictions (9) - (14) can be used together or in isolation. However, even if all the restrictions are satisfied, there may be violations of the linear ordering of the term set and loss of interpretability of functions for extreme terms, so in [17] it is proposed to use new restrictions, which can be formulated as requirements for fuzzy partition of Ruspini range of input values:

$$\forall x \in \left[\underline{x_i, \overline{x_i}}\right]: \sum_{\nu=1, \overline{V_i}} \mu_{i\nu}(x_i) = 1,$$

where  $V_i$  is the power of the term set of the variable  $x_i$ , i = 1, n. When using the Ruspini partition together with triangular membership functions, only the kernels of non-infinite fuzzy sets can be configured, since the other coefficients are related to them (Fig. 4). This decrease in the configurable coefficients reduces the accuracy of training fuzzy models [9]. In addition, the Ruspini partition is only applicable for fuzzy sets with compact carriers, such as those defined by triangular or trapezoidal membership functions.

Using such functions does not increase the size of the knowledge base, since at least one rule with a non-zero degree of execution must exist for an arbitrary input vector. In [9], the following restriction is used together with (9) - (11):

the lower and upper bounds of the core and carrier of each fuzzy term are bounded from below and from above. (15)

Compared to the fuzzy partitioning condition, these restrictions make the fuzzy models more accurate. However, the interpretability of extreme terms is not preserved during training. Another disadvantage, as with fuzzy partitioning, is the large size of the knowledge base.



Fig. 4. The Ruspini partition together with triangular membership functions



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Thus, strict restrictions in the form of Ruspini splitting preserve the transparency of the fuzzy model during training, but significantly reduce the accuracy.

The analyzed attempts to ease these restrictions increase the accuracy of training, but lead to various violations of transparency. A new approach to training membership functions is proposed below, which improves the accuracy of the Mamdani fuzzy model without compromising transparency.

#### 3. Improving the accuracy of Mamdani model training by expanding the fuzzy set carrier.

In most fuzzy Mamdani models, defuzzification is performed using the center of gravity method. For such fuzzy models, we have identified the effect of narrowing the range of output values (Fig. 5, a - the problem of narrowing theinterval), which consists in the following. The minimum value at the output of the fuzzy Mamdani model is when the degree of execution of the rule with the "Low" consequent is equal to 1, and the degree of execution of the other rules is equal to 0. In this case, the result of logical inference is found by defuzzification of the fuzzy set "Low". Similarly, the maximum output value is the result of defuzzification of the fuzzy set "High". The more blurred the fuzzy sets "Low" and "High", the further the defuzzification results are from the coordinates of the maxima of the membership functions, and, accordingly, from the required boundaries of the output values.

It is possible to eliminate the effect of narrowing the range of output values without compromising transparency by expanding the carrier of fuzzy sets (Fig. 5, b-solution method). Thus, the boundaries of the carrier of fuzzy sets of the output variable also affect the accuracy of the Mamdani model training, except for parameters listed in section 2, so it makes sense to try to improve the accuracy of fuzzy model training by adding these two parameters to the vector of controlled variables of the optimization problem (4).



Fig. 5. a) problem of narrowing the interval; b)solution method

**Example 1**. We consider the dependence (Fig. 6) of the fuel efficiency of the car (the number of miles that can be driven on one gallon of fuel — y) on the weight of the car (x1) and the model year (x2) [18]. Mamdani's fuzzy model of the  $y = f(x_1, x_2)$  dependence is shown in table 2 and table 3. The Gaussian membership function (6) is used.



Fig. 6. Of the fuel efficiency of the car



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#### Table 2. Mamdani's fuzzy model

Rule №	X1	X2	У
1	Light	New	High
2	Heavy	Old	Low
3	Light	Old	Average

#### Table 3. Mamdani's fuzzy model

Fuzzy term	b	с
Light	1613	1500
Heavy	5140	1500
Old	70	6
New	82	3
Low	9	5
Average	30	5
High	46,6	5

The testing (Fig. 7, a — the original model) shows that the fuzzy model does not work well for cars with very low and very high fuel efficiency. Expanding the fuzzy set carrier of the output variable y significantly increases the accuracy of modeling (Fig. 7, b – model with an expanded fuzzy set carrier of the output variable and Fig. 8)



Fig. 7. a) the original model; b) model with an expanded fuzzy set carrier of the output variable



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#### 4. New scheme for preserving the transparency of the Mamdani fuzzy model

To provide a small volume of the knowledge base will use fuzzy sets with non-compact carrier, for example with the Gaussian membership function (6). To prevent the effect of narrowing the range of output values let us include the boundaries of the medium fuzzy sets of the output variable to the vector of controlled variables of problem (4). To preserve the transparency of the fuzzy model, we introduce restrictions (5), (9), and (13) to the optimization problem (4). The restrictions (5) and (9) are most easily implemented by selecting the ranges of changes in the concentration coefficients (b) of the Gaussian membership function (6). In addition, we will not configure the coordinates of the maxima of the extreme terms, but set them equal to the boundaries of possible values of variables. This will protect against the loss of non-interpretability of the membership functions of extreme fuzzy terms, and also reduce the dimension of the optimization problem (4). We will only use weight coefficients for those rules whose adequacy is in doubt. The values of the other weight coefficients are equal to 1. Reducing the number of configurable parameters, in addition to reducing the optimization time, also reduces the volume of the training sample of experimental data.

**Example 2.** Let us compare the results of training the fuzzy model from the previous example using the new scheme and the standard one.

There will be 17 managed variables in the standard training scheme:

- three weight coefficients for knowledge base rules;
- seven coefficients of concentration of fuzzy term membership functions;
- seven coordinates of the maximum of fuzzy term membership functions.

We will conduct training with restrictions (9), (10) and (13).

There will be 11 managed variables in the new training scheme:

• the weight coefficient of the third rule of the knowledge base, since the adequacy of the first and second rules is obvious;

- left and right boundaries of the fuzzy set carrier of the output variable y;
- seven coefficients of the concentrations of the fuzzy term membership functions;
- maximum coordinate of the membership function of the fuzzy term "Average".

We will include cars with odd serial numbers in the training sample, and cars with even numbers in the test sample. We will conduct training using the quasi-Newtonian Broyden-Fletcher-Golfarbd-Shanno method for 15 iterations.

Distributions of errors in the test sample (statistics of 100 experiments) (Fig. 9) when optimizing from different starting points show that the training under the new scheme is on average more accurate. The membership functions and weight coefficients of the best fuzzy model rules obtained by training in both schemes are shown in Fig. 10 and in table 4. Errors in the test sample: RMSE=2.930 after training under the standard scheme and RMSE=2.884 after training under the new scheme. Despite the smaller number of configurable parameters, the new training scheme was more accurate. As for transparency, the use of the standard training scheme led to the non — interpretability of the extreme term "Heavy" (Fig. 10, a – after training according to the standard scheme). With the new training scheme, the fuzzy model remained transparent (Fig. 10, b – after training under the new scheme).





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Fig. 10

Table 4				
Rule Mo	After training under the standard	After training under the new		
Rule 312	scheme	scheme		
1	0,405	1		
2	0,994	1		
3	0,251	0,159		

Old

New

New

#### Conclusion

Old

The requirements for transparency of the fuzzy Mamdani model are formulated. Typical violations of the fuzzy model transparency that occur as a side effect of training from experimental data are identified. It is found that methods of preserving the transparency of fuzzy models form a rigid system of restrictions, which does not allow achieving high accuracy of training. It is shown that ways to improve the accuracy of training by easing restrictions somewhat violate the transparency of the fuzzy model. A new training scheme for the Mamdani fuzzy model is proposed, which differs



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from the known ones: 1) expansion of the carrier of fuzzy sets of the output variable; 2) exclusion of the maximum coordinates of functions belonging to extreme terms from the configurable parameters; 3) restriction on the linear ordering of fuzzy sets within a single term set. Computer experiments show that training using the new scheme does not violate the transparency of the fuzzy model. At the same time, the accuracy of the fuzzy model is not worse than with standard training. Due to the possibility of obtaining accurate and transparent models, the proposed training scheme is useful for identifying nonlinear dependencies using fuzzy knowledge bases in medicine, Economics, biology, sociology, and other areas where both the adequacy of the model and the meaningful interpretation of its parameters are important.

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