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# **Discrete Mathematics**

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**ABSTRACT:** Discrete mathematics is the study of mathematical structures that are discrete rather than continuous. In contrast to real numbers that vary "smoothly", discrete mathematics studies objects such as integers, graphs, and statements in logic.<sup>[1]</sup> These objects do not vary smoothly, but have distinct, separated values.<sup>[2]</sup> Discrete mathematics therefore excludes topics in "continuous mathematics" such as calculus and analysis. Discrete objects can often be counted using integers. Mathematicians say that this is the branch of mathematics dealing with countable sets<sup>[3]</sup> (sets that have the same cardinality as subsets of the natural numbers, including rational numbers but not real numbers). However, there is no exact, universally agreed, definition of the term "discrete mathematics."<sup>[4]</sup> Many times, discrete mathematics is described less by what is included than by what is excluded: continuously varying quantities and related notions.

KEYWORDS: discrete mathematics, continuous, calculus, rational, notions

#### **I.INTRODUCTION**

The set of objects studied in discrete mathematics can be finite or infinite. The term finite mathematics is sometimes applied to parts of the field of discrete mathematics that deals with finite sets, particularly those areas relevant to business.

Research in discrete mathematics increased in the latter half of the twentieth century partly due to the development of digital computers which operate in discrete steps and store data in discrete bits. Concepts and notations from discrete mathematics are useful in studying and describing objects and problems in branches of computer science, such as computer algorithms, programming languages, cryptography, automated theorem proving, and software development. In turn, computer implementations are significant in applying ideas from discrete mathematics to real-world problems, such as in operations research, game theory and graph theory.

Although the main objects of study in discrete mathematics are discrete objects, analytic methods from continuous mathematics are often employed as well. Discrete mathematics is the study of mathematical structures that are countable or otherwise distinct and separable. Examples of structures that are discrete are combinations, graphs, and logical statements. Discrete structures can be finite or infinite.

Types of Sets in Discrete Structure or Discrete Mathematics

Partially Ordered Set or POSET : Partial Ordered Set (POSET) consists of sets with three binary relations as follows.[1,2,3]

- Reflexive Relation One in which every element maps to itself.
- Anti-Symmetric Relation  $-If(a, b) \in R$  and  $(b, a) \in R$ , then a=b.
- Transitive Relation  $-If(a, b) \in R$  and  $(b, c) \in R$  then  $(a, c) \in R$ ).
  - Linearly Ordered Set : It is also known as Chain or Totally Ordered Set. It is basically a POSET in which given any pair (x, y) satisfies either x ≤ y or y ≤ x. Or we can say that if any one of the statements "x<y, y<x, x=y" is correct (This is also called as Law of Trichotomy) then it is a Linearly Ordered Set. Example A set of real numbers with a natural ordering ([R, ≤]) is a Linearly Ordered Set because firstly it is a POSET, and if we take any two real numbers e.g. (r1, r2) ∈ R then at least any one of the following three statements is always true: r1<r2 or r2<r1 or r1=r2. So, it is a Totally or Linearly Ordered Set.</li>
  - Isomorphic Ordered Sets : Let (A, ≤) and (B, ≤) be two partially ordered sets then they are said to be isomorphic if their "structures" are entirely similar. Example Let two POSETS, A = P({0, 1}) ordered by ≤ and B = {1, 2, 3, 6} ordered by division relation are Isomorphic Ordered Sets.
  - Well Ordered Set : A partially ordered set is called a Well Ordered set if every non-empty subset has a least element. Example A set of natural number and less than operation ([N, ≤]) then it is a Well ordered Set because firstly it is a POSET and if we take any two natural numbers e.g. n1 and n2 where n1≤n2. Here, n1 is the least element. So, it is a Well Ordered Set. Note –



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# Volume 11, Issue 2, February 2022

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- Any Well ordered set is totally ordered.
- Every subset of a Well ordered set is Well-ordered with the same ordering.

# **II.DISCUSSION**

Arguments are an important part of logical reasoning and philosophy. It also plays a vital role in mathematical proofs. In this article, we will throw some light on arguments in logical reasoning. Logical proofs can be proven by mathematical logic. The proof is a valid argument that determines the truth values of mathematical statements. [4,5,6] The argument is a set of statements or propositions which contains premises and conclusion. The end or last statement is called a conclusion and the rest statements are called premises.

The premises of the argument are those statements or propositions which are used to provide the support or evidence while the conclusions of an argument is that statement or proposition that simply defines that premises are intended to provide support.

Uses and Application :

- Arguments are used in computer programming.
- Arguments are used in critical thinking.
- Arguments are used to test logical ability.
- Arguments offer proof for a claim or conclusion.
- Argument mapping is useful in philosophy, management reporting, military, and intelligence analysis, and public debates.

An argument is a set of statements, including premises and the conclusion. The conclusion is derived from premises. There are two types of argument; valid argument and invalid arguments and sound and unsound. Apart from these, arguments can be deductive and inductive. There are many uses of arguments in logical reasoning and mathematical proofs.

Introduction :

Algebraic Structure : A non-empty set G equipped with 1 or more binary operations is called algebraic structure. Example –

- (N,+) where N is a set of natural numbers and
- (R, \*) R is a set of real numbers.
  - Here '\*' specifies a multiplication operation.

#### RING :

An algebraic structure that sets processing of two binary operations simultaneously is needed to form a Ring. A nonempty set R together with the operations multiplication & addition (Usually) is called a ring if :

1. (R,+) is an Abelian Group (satisfies G1, G2, G3, G4 & G5)

2. (R, \*) is a Semi Group. (satisfies G1 & G2)

3. Multiplication is distributive over addition :

(a) Left Distributive :  $a^{*}(b+c) = (a^{*}b) + (a^{*}c)$ ;  $\forall a, b, c \in \mathbb{R}$ 

(b) Right Distributive : (b+c)\*a = (b\*a) + (c\*a);  $\forall a, b, c \in R$ 

#### 1. GROUP -

An algebraic structure (G, o) where G is a non-empty set & 'o' is a binary operation defined on G is called a Group if the binary operation "o" satisfies the following properties :

G1. Closure :  $a \in G$ ,  $b \in G \Rightarrow aob \in G$ ;  $\forall a, b \in G$ 

G2. Associativity : (aob)oc = ao(boc) ;  $\forall a,b,c \in G$ .

G3. Identity Element : There exists e in G such that aoe = eoa = a ;  $\forall a \in G$  (Example – For addition, identity is 0) G4. Existence of Inverse : For each element  $a \in G$  ; there exists an inverse $(a-1)\in G$  such that : aoa-1 = a-1oa = e.

2. Abelian Group –

An algebraic structure (G, o) where G is a non-empty set & 'o' is a binary operation defined on G is called an abelian Group if it is a group (i.e., it satisfies G1, G2, G3 & G4) and additionally satisfies : G5 : Commutative:  $aob = boa \forall a, b \in G$ 



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3. Semi Group -

An algebraic structure (G, o) where G is a non-empty set & 'o' is a binary operation defined on G is called a semigroup if it satisfies only 2 properties : G1 (closure ) & G2 (Associativity). We usually write the ring structure as : (R, +, \*) or simply R. Note : Additive identity 0 is unique & is called the zero element of the Ring R. 4. Commutative Ring – (R,+,\*) is commutative : it means that the multiplication (\*) is commutative. Integral Domain : A ring (R, +, \*) is called an integral domain of : 1. (R,+,\*) is commutative. 2. (R,+,\*) is a ring with unit element. 3. It is a ring without zero divisors. 1. (R,+,\*) is commutative – means that the multiplication (\*) is commutative. 2. (R,+,\*) is a ring with a unit element – It means that there exists a unit element, say  $1 \in \mathbb{R}$ , such that :  $a^*1 = 1^*a = a \quad \forall \ a \in R[7,8,9]$ 3. R is a ring without zero divisors –  $a*b = 0 \Rightarrow a = 0 \text{ OR } b = 0 \text{ where } a, b \in \mathbb{R}$ Principal Ideal : Let (R,+,\*) is a commutative ring with identity 1. Let  $a \in \mathbb{R}$ , then the set = { ra : r  $\in \mathbb{R}$  is an ideal } called the Principal Ideal generated by a. Principal Ideal Domain (P.I.D.): A ring (R,+, \*) is called a principal ideal domain if :

- R is an integral domain.
- Every ideal in R is principal.

If every one-sided ideal of a ring is ideal, it is termed a primary ideal ring. The principal ideal domain is a principal ring with no zero divisors.

Note : integrally closed domains  $\subset$  integral domains  $\subset$  commutative rings  $\subset$  rings

#### **III.RESULTS**

Four Color Theorem and Kuratowski's Theorem in Discrete Mathematics Planar Graph :

If a graph can be drawn on the plane without crossing, it is said to be planar.

Coloring of a simple graph is the assignment of color to each vertex of the graph so that no two adjacent vertices are assigned the same color.

Bi-Partite Graphs :

A bipartite graph, also known as a bi-graph, is a set of graph vertices that has been dissected into two distinct sets, with no graph vertices in the same set being adjacent. A bipartite graph is a k-partite graph with k=2 as a particular instance. Chromatic Number :

The minimum number of colors needed to paint a graph G is called the chromatic number of G & is denoted by  $-\mu$  (G) Adjacent Regions :

An assignment of colors to the regions of a map such that adjacent regions have different colors.

A map 'M' is n – colorable if there exists a coloring of M which uses 'n' colors.

Four Color Theorem :

In 1852, Francis Guthrie, a student of Augustus De Morgan, a notable British mathematician and logician, proposed the 4-color problem. He defined the problem in terms of maps that meet specific requirements, such as not having any holes and connecting every region (e.g. country or state) so that no region exists in two or more non-contiguous sections.

Guthrie asserted that with such maps, no more than four colors would be required to color the map so that no two adjacent parts were the same color.

If the regions of a Map M are colored so that adjacent regions are different, then no more than 4 colors are



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required.[10,11,12]

Every planar graph is 4-colorable (Vertex Coloring) but when a triangle is a graph or sub-graph we need only 3 colors. Mathematicians had been attempting for years to come up with a sophisticated proof (of four color theorem) along the lines of the Six Color Theorem or the Five Color Theorem, and using the brute force method almost appeared like hacking the process.

Every planar graph can be colored in four different ways.

Vertices and edges are found in graphs. We want adjacent vertices/ regions to be of different colors.

#### How to color ?

Take any map and divide it into a set of connected regions :  $R_1, R_2 \dots R_n$  with continuous boundaries.

There must be some way to assign each region  $R_i \rightarrow$  in the set  $\{R, G, B, Y\}$ , such that if two regions  $R_i$  and  $R_j$  are "touching" (i.e. they share some nonzero length of boundary between them), they must receive different colors. Example –

1.. The four-color map is shown below :



#### A planar map

Here, as you can see, every region that touches another region has a different color than the touching one & we required a total of a maximum of four colors to color this map – Red, Green, Blue & yellow.

2. The transformation of an uncolored Map G into a colored Map is shown below -



#### Map G

Here you can see that every region that touches another region has a different color than the touching one & we required a total of maximum four colors to color this map – Red, Green, Blue & yellow.

3. The transformation of an uncolored Map H into a colored Map is shown below -

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#### Map H

Here also you can see that every region that touches another region has a different color than the touching one & we required a total of a maximum of four colors to color this map – Red, Green, Blue & yellow.

Kuratowski's Theorem :

Kuratowski established the theorem establishing a necessary and sufficient condition for planarity in 1930. The theorem states that –

"If G is non planar if and only if G contains a sub-graph that is a subdivision

of either K<sub>3,3 or K5."</sub>

To prove this theorem, we'll go through some definitions and make sure that both K3,3 and K5 are non-planar. Let's have a look at  $K_{3,3}$ .

Proposition  $1 - K_{3,3}$  is not planar.

Proof :

Now, we will prove it by contradiction.

Say, to the contrary, that  $K_{3,3}$  is planar. Then there is a plane embedding of  $K_{3,3}$  satisfying :

Then, by Euler's Formula : v - e + f = 2, where v = total vertices, e = no of edges, f = total faces.



(a) K<sub>3,3</sub> Graph

(b) K<sub>5</sub> Graph

In figure (a), the bi-partite graph : v = 6 and e = 9.

As  $K_{3,3}$  is bipartite, there are no 3-cycles in it(odd cycles can be there in it). So, each face of the embedding must be bounded by at least 4 edges from  $K_{3,3}$ . Moreover, each edge is counted twice among the boundaries for faces. Hence, we must have :  $f \le 2 * e/4$  $\Rightarrow f \le e/2$  $\Rightarrow f \le 4.5$ . Now put this data in the Euler's formula :we get : 2 = v - e + f $\Rightarrow 2 \le 6 - 9 + 4.5$  $\Rightarrow 2 \le 1.5$ , which is obviously false. So, we can say that  $K_{3,3}$  is a non-planar graph. Proposition 2 - K5 is not planar. Proof : Every planar graph must follow :  $e \le 3v - 6$  (corollary of Euler's formula) For graph (b) in the above diagram, e = 10 and v = 5. LHS : e = 10



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RHS: 3\*v - 6 = 15 - 6 = 9

 $\Rightarrow$  10  $\leq$  9, which is not true.

So, we can say that  $K_5$  is a non-planar graph.

Example :

1. Prove that : A planar graph's sub-graphs are all planar.

Proof : Let G be the graph & P be its sub-graph.

There exists a planar embedding of G, if G is planar. In the planar embedding of G, we can locate the vertices and edges of every sub-graph P of G.

This is how a planar embedding of P is created.

2. A non-planar graph's subdivisions are all non-planar.

Proof :

Assume that for G, a planar embedding of its subdivision, P, exists.

We acquire a planar embedding of G and find G planar when we remove the vertices formed in edge-subdivisions and reconstruct the original edge (without affecting the shape and position of the path). As a result, if G is non-planar, so is every subdivision (P) of G.

Hypergraph & its representation | Discrete Mathematics

A hypergraph is a graph in which hyperedges (generalized edges) can connect to a subset of vertices/nodes rather than two vertices/nodes.

The edges (also known as hyperedges) of a hypergraph are arbitrary nonempty sets of vertices. A k-hypergraph has all such hyperedges connecting exactly k vertices; a normal graph is thus a 2-hypergraph (as one edge connects 2 vertices).

Hypergraph representation :

An undirected hypergraph H is defined as a pair H = (V,E), where V is a set of items known as nodes or vertices, and E is a set of non-empty subsets of V, known as hyperedges or edges (in an undirected hypergraph).

Here, E is a subset of P(X), where P(X) is the Power Set of X.

Each hyperedge can be represented as a closed curve containing its members to create hypergraphs. [13,14,15] Example -



Hypergraph H

$$\begin{split} H(V) &= \{ A, B, C, D, E \} \\ H(E) &= \{ e1, e2, e3 \} = \{ \{A, D\}, \{D, E\}, \{A, B, C\} \} \end{split}$$

Order & Size Of Hypergraph : The order of the hypergraph = the size of the vertex set, and The size of the hypergraph = the size of the edges set. Order(H) = |H(V)|

Size(H) = |H(E)|

The above hypergraph has – Order(H) = |H(V)| = 5Size(H)= |H(E)| = 3

Hypergraph To Bi-Partite Graph : Because it is always possible (though not always convenient) to express a hypergraph by a bipartite graph,



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hypergraphs are rarely utilized. The vertex set in a bipartite graph can be divided into two subsets, P and Q, with each edge connecting a vertex in P to a vertex in Q.

We simply represent the vertices of H as vertices in Q and the hyperedges of H as vertices in P, and insert an edge (p, q) whenever s is a member of the hyperedge t in H.



A hypergraph is depicted in two ways. Five vertices on the left are connected by three hyperedges. On the right, the same five vertices are joined to new vertices(three) representing the hyperedges by ordinary edges.

Properties Of Hypergraph :

A hypergraph can have a number of different properties, including -

• Empty Hypergraph –

There are no edges in the empty hypergraph. As you see, the figure below has no edges but 5 vertices named : A B, C, D, E



Empty Hypergraph

d – Regular –

Every vertex has a degree of d, implying that it is contained in precisely d hyperedges. Example : Below, the hypergraph is 2- regular because all the vertices (A, B & C) have the same degree : 2





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2- regular hypergraph

• 2-colorable –

Its vertices can be divided into two classes, P and Q, so that each hyperedge with a cardinality of at least 2 has at least one vertex from each class.

• Non – Simple –

Has loops (hyperedges with a single vertex) or repeating edges (two or more edges with the same set of vertices) Example -

In the graph below, we can see 2 loops : e1 & e2e, so it is a non-simple hypergraph.



Non – SImple HyperGraph

• Simple –

There are no loops or repeating edges in this design.

• k -uniform –

Each hyperedge is made up of exactly k vertices. Example –

In the below hypergraph we can see that each hyperedge (e1, e2, e3, e4) consists of 2 vertices, therefore it's a 2-uniform hypergraph.



# 2-Uniform HyperGraph

k -partite –

Each hyperedge comprises exactly one vertex of each type, and the vertices are divided into k parts. Example –

In the hypergraph below, say, the vertices are partitioned into 3 parts :m (A, D),(B,E) & (D,F) . Note that each hyperedge contains only one vertex of each partition.

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#### 3-partite hypergraph

Your options after clearing GATE Examinations can be:

- 1. Go for higher studies in prestigious IITs
- 2. Apply for job roles in PSUs or MNCs
- 3. Specialize further in AI or Cybersecurity
- 4. Take up teaching roles
- 5. And much more!

A hypergraph is a graph in which hyperedges (generalized edges) can connect to a subset of vertices/nodes rather than two vertices/nodes.

The edges (also known as hyperedges) of a hypergraph are arbitrary nonempty sets of vertices. A k-hypergraph has all such hyperedges connecting exactly k vertices; a normal graph is thus a 2-hypergraph (as one edge connects 2 vertices).[16,17,18]

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Here, E is a subset of P(X), where P(X) is the Power Set of X.

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#### Hypergraph H

Order & Size Of Hypergraph :

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Order(H) = |H(V)|

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Empty Hypergraph

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2- regular hypergraph

2-colorable –

Its vertices can be divided into two classes, P and Q, so that each hyperedge with a cardinality of at least 2 has at least one vertex from each class.

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3-partite hypergraph

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- 2. Apply for job roles in PSUs or MNCs
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Types of Proofs – Predicate Logic | Discrete Mathematics

The most basic form of logic is propositional logic. Propositions, which have no variables, are the only assertions that are considered. Because there are no variables in propositions, they are either always true or always false . Example -

- 1. P: 2 + 4 = 5. (Always False) is a proposition.
- 2. Q: y \* 0 = 0. (Always true) is a proposition.

The majority of mathematical conclusions are expressed as implications: P and Q :  $P \Rightarrow Q$ We know that –

Р	Q	$P \Rightarrow Q$
Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т

Types Of Proofs :

Let's say we want to prove the implication  $P \Rightarrow Q$ . Here are a few options for you to consider.

1. Trivial Proof –

If we know Q is true, then  $P \Rightarrow Q$  is true no matter what P's truth value is.

Example –

If there are 1000 employees in a geeksforgeeks organization, then  $3^2 = 9$ .

Explanation -

Let P : There are 1000 employees in geeksforgeeks organization & Q :  $3^2 = 9$ .

We know that Q is always true and in the truth table we can see that whenever Q is true,  $P \Rightarrow Q$  is true, whatever the true value of P is.

2. Vacuous Proof -

If P is a conjunction (example :  $P = A \wedge B \wedge C$ ) of other hypotheses and we know one or more of these hypotheses is false, then P is false and so  $P \rightarrow Q$  is vacuously true regardless of the truth value of Q.

Example –

If 5 ! = 100, then 3 ! = 6.

Explanation –

Let P: 5 ! = 100, & Q: 3 ! = 6.

We know that P is always false and in the truth table we can see that whenever P is False,  $P \Rightarrow Q$  is true, whatever the truth value of Q is.

3. Direct Proof -

Assume P, then prove Q using inference rules, axioms, definitions, and logical equivalences.

Example –

For all integers p and q, if p and q are odd integers, then p + q is an even integer.

Let P denotes : p and q are odd integers

Q: p + q is an even integer

To Prove :  $P \Rightarrow Q$ 

Proof –

As p & q are odd integers, they can be represented as :

Assume : p = 2m + 1 and q = 2n + 1, where m & n are also some integers.

Then : p + q =

= (2m + 1) + (2n + 1) (Substitution Law)

= am + 2n + 2 (associative and commutative law for addition)



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= 2(m + n + 1) (distributive law) = Number divisible by 2 & hence an even number.

4. Proof By Contradiction –

We start with the assumption that the hypotheses are correct and the conclusion is incorrect, and we try to find a contradiction.

Proof by contradiction is legitimate because :

 $\neg$ (P  $\land \neg$ Q) is equivalent to P  $\Rightarrow$  Q

If we can prove that  $(P \land \neg Q)$  is false, then  $\neg (P \land \neg Q)$  is true, and the equivalent statement  $P \Rightarrow Q$  is likewise true. Example –

Let x and y be real numbers. If 5a + 25b = 156, then a or b is not an integer.

Proof -

Let P: 5a + 25b = 156 & Q: a or b is not an integer

 $\neg Q$ : a or b is an integer

So , we assume that both a and b are integers  $(\neg Q) \Rightarrow 5(a + 5b) = 156$  (distributive law)

 $\Rightarrow$  Since a and b are integers, this implies 156 is divisible by 5.

The integer 156, however, is anyway not divisible by 5. This contradiction gives the result.

It implies that  $(P \land \neg Q)$  is false as P is false

then  $\neg$  (P  $\land \neg$  Q) is true and the equivalent statement P  $\Rightarrow$  Q is likewise true.

5. Proof by Contrapositive –

We can prove  $P \Rightarrow Q$  indirectly by showing that  $\neg Q \Rightarrow \neg P$ . Assume  $\neg Q$ , and then prove  $\neg P$  using inference rules, axioms, definitions, and logical equivalences.

Example : For all integers a and b, if a\*b is even, then a is even or b is even.

Proof : We prove the contrapositive of the statement:

Let P : a\*b is even & Q : a is an even integer or b is an even integer. Then :

 $\neg P$ : a\*b is odd

 $\neg Q$ : a and b are odd integers

Say  $\neg Q$  is true, i.e., a and b are both odd integers a = 2m + 1 and b = 2n + 1; where m and n are integers. Then: a\*b=(2m + 1)(2n + 1) (by substitution) = 4mn + 2m + 2n + 1 (by associative, commutative & distributive laws) = 2(2mn + m + n) + 1 (by distributive law)

Since a\*b is twice an integer (As : 2mn + m + n is also an integer) plus 1, a\*b is odd. So it shows that  $\neg Q \Rightarrow \neg P$ . Hence  $P \Rightarrow Q$ 

#### **IV. CONCLUSION**

Discrete Mathematics | Hasse Diagrams

A Hasse diagram is a graphical representation of the relation of elements of a partially ordered set (poset) with an implied upward orientation. A point is drawn for each element of the partially ordered set (poset) and joined with the line segment according to the following rules:

- If p < q in the poset, then the point corresponding to p appears lower in the drawing than the point corresponding to q.
- The two points p and q will be joined by line segment if p is related to q.

To draw a Hasse diagram, provided set must be a poset.

A poset or partially ordered set A is a pair, (B, ) of a set B whose elements are called the vertices of A and obeys following rules: [17,18]

- 1. Reflexivity  $\rightarrow p \quad p \quad B$
- 2. Anti-symmetric  $\rightarrow p$  q and q p if p=q



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3. Transitivity  $\rightarrow$  if p q and q r then p r

Example-1: Draw Hasse diagram for ({3, 4, 12, 24, 48, 72}, /)

Explanation – According to above given question first, we have to find the poset for the divisibility. Let the set is A.

 $A=\{(3 12), (3 24), (3 48), (3 72), (4 12), (4 24), (4 48), (4 72), (12 24), (12 48), (12 72), (24 48), (24 72)\}$ 

So, now the Hasse diagram will be:



In above diagram, 3 and 4 are at same level because they are not related to each other and they are smaller than other elements in the set. The next succeeding element for 3 and 4 is 12 i.e, 12 is divisible by both 3 and 4. Then 24 is divisible by 3, 4 and 12. Hence, it is placed above 12. 24 divides both 48 and 72 but 48 does not divide 72. Hence 48 and 72 are not joined.

We can see transitivity in our diagram as the level is increasing.

Example-2: Draw Hasse diagram for (D , /)

Explanation – Here, D means set of positive integers divisors of 12. So, D  $=\{1, 2, 3, 4, 6, 12\}$ poset  $A = \{(1 \$ 2), (1 3), (1 4), (1 6), (1 12), (2 4), (2 6), (2 12), (3 6), (3 12), (4 12), (6 12)So, now the Hasse diagram will be-



In above diagram, 1 is the only element that divides all other elements and smallest. Hence, it is placed at the bottom. Then the elements in our set are 2 and 3 which do not divide each other hence they are placed at same level separately



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but divisible by 1 (both joined by 1). 4 is divisible by 1 and 2 while 6 is divisible by 1, 2 and 3 hence, 4 is joined by 2 and 6 is joined by 2 and 3. 12 is divisible by all the elements hence, joined by 4 and 6 not by all elements because we have already joined 4 and 6 with smaller elements accordingly.

For regular Hasse Diagram:

- Maximal element is an element of a POSET which is not less than any other element of the POSET. Or we can say that it is an element which is not related to any other element. Top elements of the Hasse Diagram.
- Example- In the diagram above, we can say that 4,2,3,6,1 are related to 12 (ordered by division e.g. (4,/)) but 12 is not related to any other. (As Hasse Diagram is upward directional).
- Minimal element is an element of a POSET which is not greater than any other element of the POSET. Or we can say that no other element is related to this element. Bottom elements of the Hasse Diagram.
- Example- In the diagram above, we can say that 1 is related to 2,3,4,6,12 (ordered by division e.g. (4,/)) but no element is related to 1. (As Hasse Diagram is upward directional).
- Greatest element (if it exists) is the element succeeding all other elements.
- Least element is the element that precedes all other elements.[19]

#### REFERENCES

- 1. Richard Johnsonbaugh, Discrete Mathematics, Prentice Hall, 2008.
- <sup>^</sup> Franklin, James (2017). "Discrete and continuous: a fundamental dichotomy in mathematics". Journal of Humanistic Mathematics. 7 (2): 355–378. doi:10.5642/jhummath.201702.18. S2CID 6945363. Retrieved 30 June 2021.
- 3. ^ "Discrete Structures: What is Discrete Math?". cse.buffalo.edu. Retrieved 16 November 2018.
- <sup>^</sup> Biggs, Norman L. (2002), Discrete mathematics, Oxford Science Publications (2nd ed.), The Clarendon Press Oxford University Press, p. 89, ISBN 9780198507178, MR 1078626, Discrete Mathematics is the branch of Mathematics in which we deal with questions involving finite or countably infinite sets.
- 5. ^ Hopkins, Brian, ed. (2009). Resources for Teaching Discrete Mathematics: Classroom Projects, History Modules, and Articles. Mathematical Association of America. ISBN 978-0-88385-184-5.
- 6. <sup>^</sup> Kurgalin, Sergei; Borzunov, Sergei (2020). "The Discrete Math Workbook". Texts in Computer Science. doi:10.1007/978-3-030-42221-9. ISBN 978-3-030-42220-2. ISSN 1868-0941.
- 7. ^ Levasseur, Ken; Doerr, Al. Applied Discrete Structures. p. 8.
- <sup>A</sup> Geoffrey Howson, Albert, ed. (1988). Mathematics as a Service Subject. Cambridge University Press. pp. 77– 78. ISBN 978-0-521-35395-3.
- <sup>^</sup> Rosenstein, Joseph G. Discrete Mathematics in the Schools. American Mathematical Society. p. 323. ISBN 978-0-8218-8578-9.
- 10. ^ "UCSMP". uchicago.edu.
- 11. <sup>^</sup> Troelstra, A.S.; Schwichtenberg, H. (2000-07-27). Basic Proof Theory. Cambridge University Press. p. 186. ISBN 978-0-521-77911-1.
- 12. ^ Buss, Samuel R. (1998). Handbook of Proof Theory. Elsevier. p. 13. ISBN 978-0-444-89840-1.
- <sup>A</sup> Baader, Franz; Brewka, Gerhard; Eiter, Thomas (2001-10-16). KI 2001: Advances in Artificial Intelligence: Joint German/Austrian Conference on AI, Vienna, Austria, September 19-21, 2001. Proceedings. Springer. p. 325. ISBN 978-3-540-42612-7.
- <sup>A</sup> Brotherston, J.; Bornat, R.; Calcagno, C. (January 2008). "Cyclic proofs of program termination in separation logic". ACM SIGPLAN Notices. 43 (1): 101–112. doi:10.1145/1328897.1328453.
- 15. <sup>A</sup> Mohar, Bojan; Thomassen, Carsten (2001). Graphs on Surfaces. Johns Hopkins University Press. ISBN 978-0-8018-6689-0. OCLC 45102952.
- 16. <sup>^</sup> Jump up to:<sup>a b</sup> Wilson, Robin (2002). Four Colors Suffice. London: Penguin Books. ISBN 978-0-691-11533-7.
- 17. ^ Hodges, Andrew (1992). Alan Turing: The Enigma. Random House.
- <sup>A</sup> Hodkinson, Trevor R.; Parnell, John A. N. (2007). Reconstruction the Tree of Life: Taxonomy And Systematics of Large And Species Rich Taxa. CRC Press. p. 97. ISBN 978-0-8493-9579-6.
- 19. ^ "Millennium Prize Problems". 2000-05-24. Retrieved 2008-01-12.





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