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Stress Effect in Two Dimensional Rectangular Plate with Third Kind Boundary Conditions

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ABSTRACT: The present work is based on the temperature, displacement and stress analysis in a two dimensional thin rectangular plate occupying the space $D: -a \leq x \leq a; -b \leq y \leq b$; with the third kind boundaries and initial conditions. The solution of the heat equation is obtained by applying Marchi – Fasulo transform and Laplace transform techniques. The obtained results of the numerical solutions are depicted graphically.

KEYWORDS AND PHRASES: two dimensional rectangular plate, stress, integral transform.

2001 Mathematics Subject Classification: 74J25, 74H99, 74D99

I. INTRODUCTION

The first attempt to examine the inertia effects in a transient thermoelastic problem is apparently due to Danilovskaya [1] Sternberg and Chakravorthy [2] obtained the missing displacements in the closed form as in [1] and showed that the dynamic effects induced by gradual heating or cooling of the boundary are substantially different from those encountered in [3], even at extremely high, but finite time gradients of the surface temperature.

The inverse problem however have been treated only in a few papers quoted below: Cialkowski and Grysa [4], Grysa, Cialkowski and Kaminski [5], Grysa, Kozlowski [6],[7] and Kumar and Wankhede [8] have investigated one dimensional transient thermoelastic problems and derived the heating temperature and heat flux on the surface of an isotropic infinite slab. Noda, Ashida and Tsuji [9] discussed an analytical method for an inverse problem of three dimensional transient thermoelasticity in a transversely isotropic solid by applying Laplace and Fourier transforms with newly designed potential function and illustrated the practical applicability of the method in engineering problems of great importance.

In the present paper, authors try to determine the temperature, displacement and thermal stress at any point of a thin rectangular plate occupying the region $D: -a \leq x \leq a; -b \leq y \leq b$; with certain boundary conditions. Here Marchi – Fasulo transform and Laplace transform techniques have been used to find the solution of the problem.

II. STATEMENT OF THE PROBLEM

Consider a thin rectangular plate occupying the space $D: -a \leq x \leq a, -b \leq y \leq b$. The displacement components u_x, u_y in the X, Y direction respectively are in the integral form as

$$u_x = \int_{-a}^a \frac{1}{E} \left(\frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial x^2} + \alpha T \right) dx \quad (1)$$

$$u_y = \int_{-b}^b \frac{1}{E} \left(\frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2} + \alpha T \right) dy \quad (2)$$

where E, ν and α are the Young's modulus, Poisson's ratio and the linear coefficient of thermal expansion of the material of the plate respectively, $U(x, y, t)$ is the Airy's stress function which satisfies the differential equation namely,



$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 U(x, y, t) = -\alpha E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) T(x, y, t). \tag{3}$$

where $T(x, y, t)$ denotes the temperature of thin rectangular plate satisfying the following differential equation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{k} \frac{\partial T}{\partial t} \tag{4}$$

where k is thermal diffusivity of the material subject to initial conditions

$$T(x, y, 0) = 0. \tag{5}$$

The boundary conditions are

$$\left[T(x, y, t) + k_1 \frac{\partial T(x, y, t)}{\partial x} \right]_{x=a} = F_1(y, t) \tag{6}$$

$$\left[T(x, y, t) + k_2 \frac{\partial T(x, y, t)}{\partial x} \right]_{x=-a} = F_2(y, t) \tag{7}$$

$$\left[T(x, y, t) + k_3 \frac{\partial T(x, y, t)}{\partial y} \right]_{y=b} = F_3(x, t) \tag{8}$$

$$\left[T(x, y, t) + k_4 \frac{\partial T(x, y, t)}{\partial y} \right]_{y=-b} = F_4(x, t) \tag{9}$$

The stresses components in terms of $U(x, y, z, t)$ are given by

$$\sigma_{xx} = \left(\frac{\partial^2 U}{\partial y^2} \right) \tag{10}$$

$$\sigma_{yy} = \left(\frac{\partial^2 U}{\partial x^2} \right) \tag{11}$$

The equations (1) to (11) constitute the mathematical formulation of the problem under consideration.

III SOLUTION OF THE PROBLEM

By applying finite Marchi – Fasulo transform and Laplace transform to the equations (4) to (9), and then their inversion, one obtain

$$\begin{aligned} T(x, y, t) = & \frac{k}{c^2} \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\tau_1(t) - \tau_2(t) \right] \\ & + 2k\pi \sum_{m,n,\zeta=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{\zeta}{\cos \zeta\pi} \right] \left[\frac{\tau_3(t) - \tau_4(t)}{1 + (c\zeta\pi)^2} \right] \\ & - \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] A_3(m, n, t) \end{aligned} \tag{12}$$



where

$$\tau_1(t) = \int_0^t [\bar{f}(m, n, t-u) - A_1(m, n, t-u)] e^{ku \left[\frac{1-c^2q^2}{c^2} \right]} du$$

$$\tau_2(t) = \int_0^t [\bar{g}(m, n, t-u) - A_2(m, n, t-u)] e^{ku \left[\frac{1-c^2q^2}{c^2} \right]} du$$

$$\tau_3(t) = \int_0^t [\bar{f}(m, n, t-u) - A_1(m, n, t-u)] e^{-ku[q^2+(\sigma\tau)^2]} du$$

$$\tau_4(t) = \int_0^t [\bar{g}(m, n, t-u) - A_2(m, n, t-u)] e^{-ku[q^2+(\sigma\tau)^2]} du$$

$$A_1(m, n, t) = L^{-1} \left[\left(\chi + c \frac{d\chi}{dz} \right) \right]$$

$$A_2(m, n, t) = L^{-1} \left[\left(\chi + c \frac{d\chi}{dz} \right) \right]$$

$$A_3(m, n, t) = L^{-1} [\chi]$$

Here $\bar{f}(m, n, t)$ and $\bar{g}(m, n, t)$ denote the Marchi – Fasulo transforms of $\bar{f}(m, y, t)$ and $\bar{g}(m, y, t)$ respectively. $\bar{f}(m, y, t)$ $\bar{g}(m, y, t)$ denote the finite Marchi – Fasulo transform of $f(x, y, t)$ and $g(x, y, t)$ respectively.

$$\bar{f}(m, n, t) = \int_{-b}^b \bar{f}(m, y, t) P_n(y) dy$$

$$\bar{g}(m, n, t) = \int_{-b}^b \bar{g}(m, y, t) P_n(y) dy$$

$$\lambda_n = \int_{-b}^b P_n^2(y) dy$$

$$P_n(y) = Q_n \cos(a_n y) - W_n \sin(a_n y)$$

$$Q_n = a_n (\alpha_3 + \alpha_4) \cos(a_n b) + (\beta_3 - \beta_4) \sin(a_n b)$$



$$W_n = (\beta_3 + \beta_4) \cos(a_n b) + (\alpha_4 - \alpha_3) a_n \sin a_n b$$

Equation (12) is the desired solution of the given problem with $\beta_3 = \beta_4 = 1$, $\alpha_3 = k_3$, $\alpha_4 = k_4$.

IV. DETERMINATION OF AIRYS STRESS FUNCTION

Substituting the values of $T(x, y, t)$ from equation (12) in equation (3) one obtains

$$\begin{aligned}
 U(x, y, t) = & \frac{\alpha E k}{c^2} \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{[\tau_1(t) - \tau_2(t)]}{a_m^2 + a_n^2 - 1/c^2} \right] \\
 & + 2\alpha E k \pi \sum_{m,n,\zeta=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{\zeta}{\cos \zeta \pi} \right] \left[\frac{1}{1 + (c \zeta \pi)^2} \right] \\
 & \times \left[\frac{\tau_3(t) - \tau_4(t)}{a_m^2 + a_n^2 + (\zeta \pi)^2} \right] - \alpha E \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{A_3(m, n, t)}{a_m^2 + a_n^2 - l_0} \right] \quad (13)
 \end{aligned}$$

V. DETERMINATION OF DISPLACEMENT COMPONENTS

Substituting the value of $U(x, y, t)$ in the equation (1) and (2) one obtains

$$\begin{aligned}
 u_x = & \frac{\alpha k}{c^2} \sum_{m,n=1}^{\infty} \left[\frac{(k_1 + k_2) \sin 2a_m a}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{(1 + \nu) a_m^2}{a_m^2 + a_n^2 - 1/c^2} \right] \\
 & \times [\tau_1(t) - \tau_2(t)] \\
 & + 2\alpha k \pi \sum_{m,n,\zeta=1}^{\infty} \left[\frac{(k_1 + k_2) \sin 2a_m a}{\lambda_m} \right] \left[\frac{\zeta}{\cos \zeta \pi} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{(1 + \nu) a_m^2}{a_m^2 + a_n^2 + (\zeta \pi)^2} \right] \\
 & \times \left[\frac{\tau_3(t) - \tau_4(t)}{1 + (c \zeta \pi)^2} \right] \\
 & - \alpha \sum_{m,n=1}^{\infty} \left[\frac{(k_1 + k_2) \sin 2a_m a}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{(1 + \nu) a_m^2}{a_m^2 + a_n^2 - l_0} \right] A_3(m, n, t), \quad (14)
 \end{aligned}$$

$$\begin{aligned}
 u_y = & \frac{\alpha k}{c^2} \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{(k_3 + k_4) \sin 2a_n b}{\lambda_n} \right] \left[\frac{(1 + \nu) a_n^2}{a_m^2 + a_n^2 - 1/c^2} \right] \\
 & \times \left[\tau_1(t) - \tau_2(t) \right] \\
 & + 2\alpha k \pi \sum_{m,n,\zeta=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{(k_3 + k_4) \sin 2a_n b}{\lambda_n} \right] \left[\frac{\zeta}{\cos \zeta \pi} \right] \\
 & \left[\frac{(1 + \nu) a_n^2}{a_m^2 + a_n^2 + (\zeta \pi)^2} \right] \left[\frac{\tau_3(t) - \tau_4(t)}{1 + (c \zeta \pi)^2} \right] \\
 & - \alpha \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{(k_3 + k_4) \sin 2a_n b}{\lambda_n} \right] \left[\frac{(1 + \nu) a_n^2}{a_m^2 + a_n^2 - l_0} \right] A_3(m, n, t), \tag{15}
 \end{aligned}$$

VI. DETERMINATION OF STRESS FUNCTION

Substituting value of (13) in equations (10) and (11) one obtains

$$\begin{aligned}
 \sigma_{xx} = & \frac{\alpha E k}{c^2} \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{-a_n^2 + 1/c^2}{a_m^2 + a_n^2 - 1/c^2} \right] \left[\tau_1(t) - \tau_2(t) \right] \\
 & + 2\alpha E k \pi \sum_{m,n,\zeta=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{\zeta}{\cos(\zeta \pi)} \right] \left[\frac{-a_n^2 - (\zeta \pi)^2}{a_m^2 + a_n^2 + (\zeta \pi)^2} \right] \\
 & \left[\frac{\tau_3(t) - \tau_4(t)}{1 + (c \zeta \pi)^2} \right] - \alpha E \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{l_0 - a_n^2}{a_m^2 + a_n^2 - l_0} \right] \\
 & [A_3(m, n, t)], \tag{16}
 \end{aligned}$$

$$\begin{aligned}
 \sigma_{yy} = & \left(\frac{\alpha E k}{c^2} \right) \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{1/c^2 - a_m^2}{a_m^2 + a_n^2 - 1/c^2} \right] \left[\tau_1(t) - \tau_2(t) \right] \\
 & + 2\alpha E k \pi \sum_{m,n,\zeta=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{\zeta}{\cos(\zeta \pi)} \right] \left[\frac{-(\zeta \pi)^2 - a_m^2}{a_m^2 + a_n^2 + (\zeta \pi)^2} \right] \\
 & \left[\frac{\tau_3(t) - \tau_4(t)}{1 + (c \zeta \pi)^2} \right]
 \end{aligned}$$



$$-\alpha E \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m} \right] \left[\frac{P_n(y)}{\lambda_n} \right] \left[\frac{l_0 - a_m^2}{a_m^2 + a_n^2 - l_0} \right] A_3(m,n,t), \tag{17}$$

VII. SPECIAL CASE AND NUMERICAL RESULTS

Set $f(x, y, t) = (1 - e^{-t})(x + a)^2(x - a)^2(y + b)^2(y - b)^2$,

$g(x, y, t) = (1 - e^{-t})(x + a)^2(x - a)^2(y + b)^2(y - b)^2 e^h$,

$\delta = \frac{8(k_1 + k_2)k\pi}{h^2}$, $a = 0.75$, $k = 0.86$, $b = 0.75$, $h = 0.25$, $t = 1$ in the equation (12) to obtain

$$\begin{aligned} \frac{T(x, y, t)}{\delta} &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{\eta=1}^{\infty} (-1)^{(\eta+1/2)} \left(\eta + \frac{1}{2} \right) \left(\frac{P_n(x)}{\mu_n} \right) \left(\frac{P_m(y)}{\lambda_m} \right) \left(\frac{1}{1 - q^2} \right) \\ &\times \left[\frac{a_n \cos^2(a_n) - \cos(a_n) \sin(a_n)}{a_n^2} \right] \times [e - 1] \\ &\times \int_0^t (1 - e^{-t'}) e^{-0.86 \left(q^2 + \left(\eta + \frac{1}{2} \right)^2 \pi^2 \right) (t-t')} dt' \end{aligned} \tag{18}$$

VIII. CONCLUSION

The solution of temperature distribution, displacements, and thermal stresses on the upper plane surface of a rectangular plate have been obtained successfully, when the third kind boundary conditions are applied with the aid of finite Marchi-Fasulo transform and Laplace transform techniques. The expressions are represented graphically. The results are obtained in the form of infinite series. It is observed that as x increases the temperature gradually decreases. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

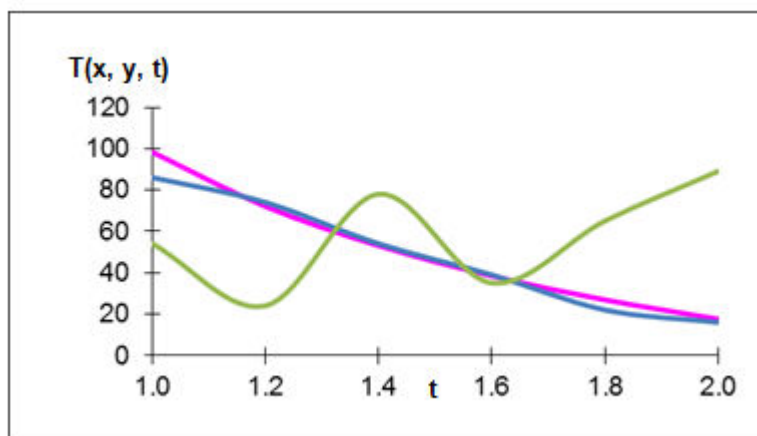


Figure 1: T(x, y, t) versus t for different values of x

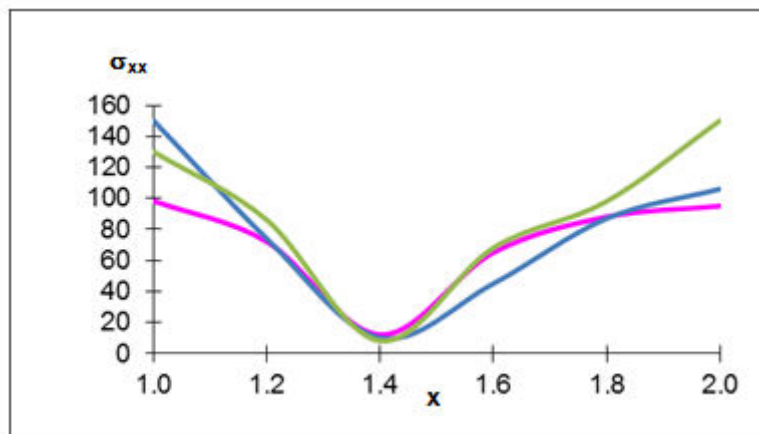


Figure 2: σ_{xx} versus x for different values of t

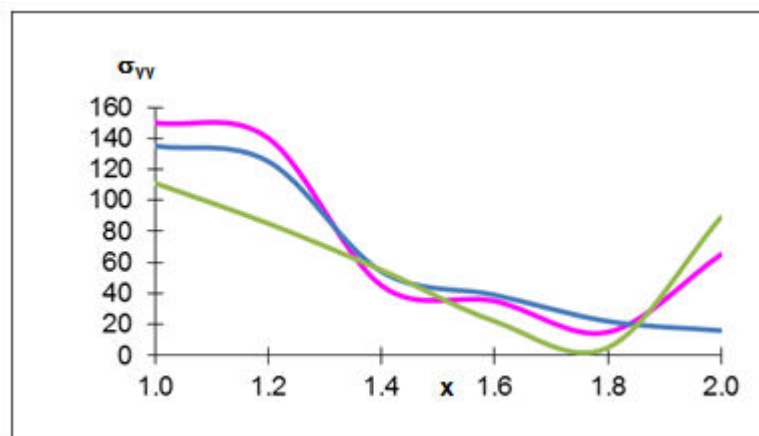


Figure 3: σ_{yy} versus x for different values of t

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