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Application of Ordinary Differential Equations in Science & Various Engineering Fields with Real Life Applications

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ABSTRACT: In this paper, we will discuss about various applications of Differential Equation in different engineering fields. Differential Equation applications have great significance in academic research work and real life. We will discuss about few real life applications and the way to solve Differential Equations by using various methods

KEYWORDS: Differential Equation, First order, First degree, VSDE, Liner Differential Equation, projectile, Range & Time of flight, Velocity, Acceleration.

I. INTRODUCTION

An equation, carrying various differential Coefficients, is a **Differential Equation** which relates one or more unknown functions and their derivatives. In applications, the functions generally represent physical quantities, the derivatives represent their rates of change, and the differential equation defines a relationship between the two. Such relations are common; therefore, differential equations play a prominent role in many disciplines Science & Engineering, Economics, Biology etc..

Sir Isaac Newton & Leibnitz were first to use differential equations with the invention of Calculus.

Isaac Newton listed three kinds of differential equations:

$$\frac{dy}{dx} = f(x)$$

$$\frac{dy}{dx} = f(x, y)$$

$$u \cdot \frac{\partial y}{\partial u} + v \cdot \frac{\partial y}{\partial v} = y$$

In all these cases, y is an unknown function of x (or of u and v), and f is a given function

In the year 1665, Jacob Bernoulli greatly contributed to linear differential Equation which was named as Bernoulli differential equation and popularly known as Reducible to Linear Differential Equation This is an ordinary differential equation of the form

$$\frac{dy}{dx} + P(x) \cdot y = Q(x) \cdot y^n$$

TYPES:

Differential equations can be divided into several types. Apart from describing the properties of the equation itself, these classes of differential equations can help inform the choice of approach to a solution. Commonly used distinctions include whether the equation is ordinary or partial, linear or non-linear, and homogeneous or heterogeneous. This list is far from exhaustive; there are many other properties and subclasses of differential equations which can be very useful in specific contexts.

1. Ordinary differential equations

An ordinary differential equation (ODE) is an equation containing an unknown function of one real or complex variable x , its derivatives, and some given functions of x . The unknown function is generally

represented by a variable (often denoted y), which, therefore, depends on x . Thus x is often called the independent variable of the equation. The term "ordinary" is used in contrast with the term partial differential equation, which may be with respect to more than one independent variable.

Linear differential equations are the differential equations that are linear in the unknown function and its derivatives. Their theory is well developed, and in many cases one may express their solutions in terms of integrals.

Most ODEs that are encountered in physics are linear. Therefore, most special functions may be defined as solutions of linear differential equations.

As, in general, the solutions of a differential equation cannot be expressed by a closed-form expression, numerical methods are commonly used for solving differential equations on a computer.

2. Partial differential equations

A partial differential equation (*PDE*) is a differential equation that contains unknown multivariable functions and their partial derivatives. (This is in contrast to ordinary differential equations, which deal with functions of a single variable and their derivatives.) PDEs are used to formulate problems involving functions of several variables, and are either solved in closed form, or used to create a relevant computer model.

PDEs can be used to describe a wide variety of phenomena in nature such as sound, heat, electrostatics, electrodynamics, fluid flow, elasticity, or quantum mechanics. These seemingly distinct physical phenomena can be formalized similarly in terms of PDEs. Just as ordinary differential equations often model one-dimensional dynamical systems, partial differential equations often model multidimensional systems. Stochastic partial differential equations generalize partial differential equations for modeling randomness.

3. Non-linear differential equations

A **non-linear differential equation** is a differential equation that is not a linear equation in the unknown function and its derivatives (the linearity or non-linearity in the arguments of the function are not considered here). There are very few methods of solving nonlinear differential equations exactly; those that are known typically depend on the equation having particular symmetries. Nonlinear differential equations can exhibit very complicated behavior over extended time intervals, characteristic of chaos. Even the fundamental questions of existence, uniqueness, and extendibility of solutions for nonlinear differential equations, and well-posedness of initial and boundary value problems for nonlinear PDEs are hard problems and their resolution in special cases is considered to be a significant advance in the mathematical theory. However, if the differential equation is a correctly formulated representation of a meaningful physical process, then one expects it to have a solution.

Linear differential equations frequently appear as approximations to nonlinear equations. These approximations are only valid under restricted conditions. For example, the harmonic oscillator equation is an approximation to the nonlinear pendulum equation that is valid for small amplitude oscillations.

Order & Degree of a Differential Equation

Differential equations are described by their order and Degree. Order of a Differential Equation is determined by the order of highest derivatives, present in the equation. An equation containing only first derivatives is a first-order differential equation, an equation containing the second derivative is a second-order differential equation, and so on. Differential equations that describe natural phenomena almost always have only first and second order derivatives in them, but there are some exceptions, such as the thin film equation, which is a fourth order partial differential equation

The degree of a Differential Equation is described as the indices of the highest order derivative, present in the equation, provided the equation does not carry any fractional indices.

As an example, a second order and first degree differential equation is given by

$$x. \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

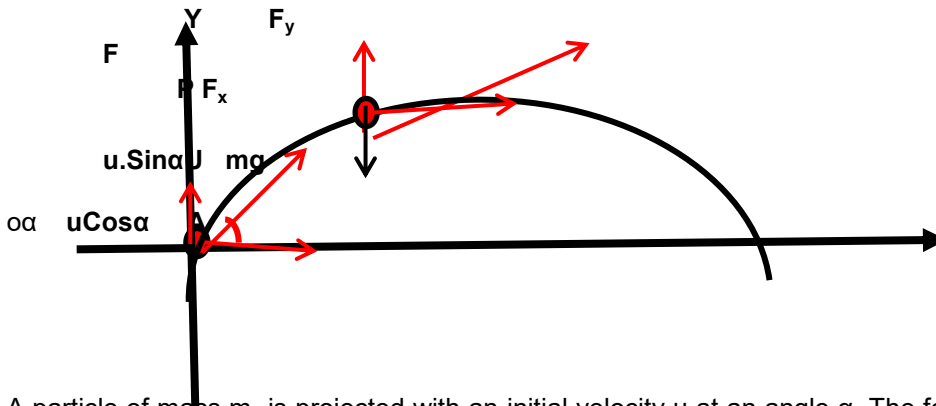
As an example for first order and second degree differential equation is given by

$$x \cdot \left\{ \frac{dy}{dx} \right\}^2 + y = 0$$

APPLICATIONS OF DIFFERENTIAL EQUATIONS IN SCIENCE AND ENGINEERING FIELD

1. Differential Equations in Projectile Motion

A particle is projected with an initial velocity u at an angle α . Let us consider a point $P(x,y)$ on the parabolic trajectory of the projectile.



A particle of mass m is projected with an initial velocity u at an angle α . The force F , responsible for motion of the particle is reduced into its components F_x & F_y .

At any point $p(x,y)$ on the trajectory

$$F_x = 0 \quad \text{and} \quad F_y = -mg$$

$$\Rightarrow \frac{d^2x}{dt^2} = 0 \quad \text{and} \quad m \cdot \frac{d^2y}{dt^2} = -m \cdot g$$

Both are Variable Seperable type First order First degree Differential Equations.

On Integration, we get, $\frac{dx}{dt} = C_1$ and $\frac{dy}{dt} = -g \cdot t + C_2$

$$u \cdot \cos \alpha = C_1 \quad \text{and} \quad u \cdot \sin \alpha = -g \cdot t + C_2$$

Initially at $t=0$, gives results as

$$u \cdot \cos \alpha = 0 \quad \text{and} \quad u \cdot \sin \alpha = -g \cdot t$$

Hence we get, $\frac{dx}{dt} = u \cdot \cos \alpha$ and $\frac{dy}{dt} = u \cdot \sin \alpha - g \cdot t$

Both are Variable Seperable type First order First degree Differential Equations.

$$\Rightarrow x = u \cdot \cos \alpha \cdot t + C_3 \quad \text{and} \quad y = u \cdot \sin \alpha \cdot t - g \cdot \frac{t^2}{2} + C_4$$

$$\Rightarrow \text{At } t=0, \text{ particle is at origin, means } u=0, x=y=0$$

$$\Rightarrow C_3 = 0, C_4 = 0$$

$$\Rightarrow x = u \cdot \cos \alpha \cdot t \quad \text{and} \quad y = u \cdot \sin \alpha \cdot t - g \cdot \frac{t^2}{2}$$

Let T be the Time of flight of Projectile, taken to reach at A on x-axis, So putting $y=0$, and $t = T$, we get

$$\Rightarrow u \sin \alpha \cdot T - g \cdot \frac{T^2}{2} = 0 \Rightarrow T = \frac{2u \sin \alpha}{g}$$

Let R be the Range of Projectile to Reach at A on x-axis then

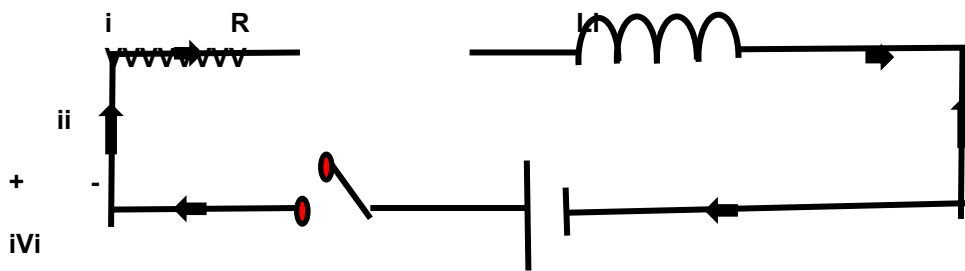
$$\text{Displacement } R = x = u \cos \alpha \cdot T \text{ So } R = u \cos \alpha \cdot \frac{2u \sin \alpha}{g}$$

$$\text{Hence, } R = \frac{u^2 \sin 2\alpha}{g}$$

This could be used in launching a Rocket for propagating a satellite or a missile to destroy targets in war zone.

2. Current at any Time in an R-L Circuit

In a network R-L circuit, with resistance $R \Omega$ & Inductance L Henry, a constant voltage V Volts is applied to the circuit at $t=0$ by closing switch. Find current at any time t Sec flowing in the circuit.



Applying Kirchhoff's Voltage Law, we get

$$L \cdot \frac{di}{dt} + i \cdot R = V$$

$$\frac{di}{dt} + i \cdot \frac{R}{L} = \frac{V}{L}$$

This is Linear Differential Equation

$$\text{Here } P = \frac{R}{L}, Q = \frac{V}{L}$$

$$I.F = e^{\int P \cdot dt} = e^{\int \frac{R}{L} dt} = e^{\frac{R \cdot t}{L}}$$

Solution is given by—

$$y \cdot I.F = \int Q \cdot I.F dx + C$$

$$i \cdot e^{\frac{R \cdot t}{L}} = \int \frac{V}{L} \cdot e^{\frac{R \cdot t}{L}} dt + C$$



$$i. e^{\frac{Rt}{L}} = \frac{V}{R} \cdot e^{\frac{Rt}{L}} + C$$

$$i. = \frac{V}{R} + C. e^{-\frac{Rt}{L}}$$

At $t = 0$ Sec, There is no current in the circuit, means $i = 0$

$$\Rightarrow C = -\frac{V}{R} \Rightarrow i = \frac{V}{R} \cdot \{ 1 - e^{-\frac{Rt}{L}} \}$$

This Formula gives the value of current flowing in the circuit at any time t .

3. Velocity and Acceleration of Particle at any time t

The Acceleration of a particle is given by $2t+3$. Find the velocity & distance travelled by particle in t seconds.

$$\text{Acceleration} = \frac{d^2s}{dt^2} = 2t + 3$$

This is Variable Seperable type First order First degree Differential Equation.

$$\int \frac{d^2s}{dt^2} \cdot dt = \int (2t + 3) \cdot dt$$

$$\Rightarrow \text{Velocity} = V = \frac{ds}{dt} = t^2 + 3t + C$$

At $t = 0$, $v = 0$ means $C = 0$

$$\text{So Velocity } V = t^2 + 3t$$

$$\text{Now, } \int \frac{ds}{dt} \cdot dt = \int (t^2 + 3t) \cdot dt$$

$$\text{Distance } S = \frac{t^3}{3} + 3 \cdot \frac{t^2}{2} + K$$

$$\text{At } t = 0, S = 0 \text{ means } K = 0$$

$$\text{Distance } S = \frac{t^3}{3} + 3 \cdot \frac{t^2}{2}$$

4. Finding an Equation of Curve if slope is given

Find the Equation of the Curve whose slope is given by $(2a \cdot x + b)$ and passes through (c,d) .

$$\text{Slope of a Curve} = \frac{dy}{dx} = 2a \cdot x + b$$

This is Variable Seperable type First order First degree Differential Equation.

$$\Rightarrow \int dy = \int (2a \cdot x + b) dx$$

$$\Rightarrow y = 2a \cdot \frac{x^2}{2} + b \cdot x + C = a \cdot x^2 + bx + C$$



As the curve passes through (c, d), it must satisfy the equation of the curve.

$$\Rightarrow d = a.c^2 + b.c + C \Rightarrow C = d - a.c^2 - b.c = K \text{ (say)}$$

Therefore, equation of the Curve is $y = a.x^2 + bx + K$

II. CONCLUSION

Through this paper, I present the applications of Ordinary Differential Equations in Science and Engineering with real life applications. In this paper, I have tried to explain the applications of VSDE & LDE with the help of few examples, connected to real life situations. It goes without saying that Ordinary Differential Equations have tremendous applications in Science & Engineering field.

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