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On Absolute Summability Factors of Infinite Series

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ABSTRACT: In mathematics, summation is the addition of a sequence of any kind of numbers, called addends or summands; the result is their sum or total. Beside numbers, other types of values can be summed as well: functions, vectors, matrices, polynomials and, in general, elements of any type of mathematical objects on which an operation denoted "+" is defined.

Summations of infinite sequences are called series. They involve the concept of limit, and are not considered in this article.

The summation of an explicit sequence is denoted as a succession of additions. For example, summation of [1, 2, 4, 2] is denoted $1 + 2 + 4 + 2$, and results in 9, that is, $1 + 2 + 4 + 2 = 9$. Because addition is associative and commutative, there is no need of parentheses, and the result is the same irrespective of the order of the summands. Summation of a sequence of only one element results in this element itself. Summation of an empty sequence (a sequence with no elements), by convention, results in 0.

Very often, the elements of a sequence are defined, through a regular pattern, as a function of their place in the sequence. For simple patterns, summation of long sequences may be represented with most summands replaced by ellipses. For example, summation of the first 100 natural numbers may be written as $1 + 2 + 3 + 4 + \dots + 99 + 100$. Otherwise, summation is denoted by using Σ notation, where Σ is an enlarged capital Greek letter sigma. For example, the sum of the first n natural numbers can be denoted as

For long summations, and summations of variable length (defined with ellipses or Σ notation), it is a common problem to find closed-form expressions for the result

KEYWORDS: summability factors, infinite series, sequence explicit, convention, cumulative, summands

I. INTRODUCTION

Capital-sigma notation

This is defined as

where i is the index of summation; a_i is an indexed variable representing each term of the sum; m is the lower bound of summation, and n is the upper bound of summation. The " $i = m$ " under the summation symbol means that the index i starts out equal to m . The index, i , is incremented by one for each successive term, stopping when $i = n$.¹

This is read as "sum of a_i , from $i = m$ to n ".

Here is an example showing the summation of squares:

In general, while any variable can be used as the index of summation (provided that no ambiguity is incurred), some of the most common ones include letters such as i , j , and k ; the latter is also often used for the upper bound of a summation.

Alternatively, index and bounds of summation are sometimes omitted from the definition of summation if the context is sufficiently clear. This applies particularly when the index runs from 1 to n .^[1] For example, one might write that:²

Generalizations of this notation are often used, in which an arbitrary logical condition is supplied, and the sum is intended to be taken over all values satisfying the condition.³

In mathematics, ancient Egyptian multiplication (also known as Egyptian multiplication, Ethiopian multiplication, Russian multiplication, or peasant multiplication), one of two multiplication methods used by scribes, is a systematic method for multiplying two numbers that does not require the multiplication table, only the ability to



multiply and divide by 2, and to add. It decomposes one of the multiplicands (preferably the smaller) into a set of numbers of powers of two and then creates a table of doublings of the second multiplicand by every value of the set which is summed up to give result of multiplication.⁴

This method may be called mediation and duplation, where mediation means halving one number and duplation means doubling the other number. It is still used in some areas.^[1]

The second Egyptian multiplication and division technique was known from the hieratic Moscow and Rhind Mathematical Papyri written in the seventeenth century B.C. by the scribe Ahmes.^[2]

Although in ancient Egypt the concept of base 2 did not exist, the algorithm is essentially the same algorithm as long multiplication after the multiplier and multiplicand are converted to binary. The method as interpreted by conversion to binary is therefore still in wide use today as implemented by binary multiplier circuits in modern computer processors.^[1]

It is possible to sum fewer than 2 numbers: If the summation has one summand, then the evaluated sum⁵

If the summation has no summands, then the evaluated sum is zero, because zero is the identity for addition. This is known as the empty sum.

These degenerate cases are usually only used when the summation notation gives a degenerate result in a special case.

For example, if $\sum_{i=1}^n$ in the definition above, then there is only one term in the sum; if $\sum_{i=1}^0$, then there is none.

In 1675, Gottfried Wilhelm Leibniz, in a letter to Henry Oldenburg, suggests the symbol \int to mark the sum of differentials (Latin: calculus summatorius), hence the S-shape.^{[4][5][6]} The renaming of this symbol to integral arose later in exchanges with Johann Bernoulli.^[6] In 1755, the summation symbol Σ is attested in Leonhard Euler's *Institutiones calculi differentialis*.^{[7][8]} Euler uses the symbol in expressions like:^{5,6}

- In 1772, usage of Σ and Σ^n is attested by Lagrange.^{[7][9]}
- In 1823, the capital letter S is attested as a summation symbol for series. This usage was apparently widespread.^[7]
- In 1829, the summation symbol Σ is attested by Fourier and C. G. J. Jacobi.^[7] Fourier's use includes lower and upper bounds, for example.^{[10][11]}

There are many sets that, under the operation of multiplication, satisfy the axioms that define group structure. These axioms are closure, associativity, and the inclusion of an identity element and inverses.

A simple example is the set of non-zero rational numbers. Here we have identity 1, as opposed to groups under addition where the identity is typically 0. Note that with the rationals, we must exclude zero because, under multiplication, it does not have an inverse: there is no rational number that can be multiplied by zero to result in 1. In this example, we have an abelian group, but that is not always the case.^{8,9}

To see this, consider the set of invertible square matrices of a given dimension over a given field. Here, it is straightforward to verify closure, associativity, and inclusion of identity (the identity matrix) and inverses. However, matrix multiplication is not commutative, which shows that this group is non-abelian.¹⁰

Another fact worth noticing is that the integers under multiplication do not form a group—even if we exclude zero. This is easily seen by the nonexistence of an inverse for all elements other than 1 and -1 .¹¹

II. DISCUSSION

In mathematics, an iterated binary operation is an extension of a binary operation on a set S to a function on finite sequences of elements of S through repeated application.^[1] Common examples include the extension of the addition operation to the summation operation, and the extension of the multiplication operation to the product operation. Other operations, e.g., the set-theoretic operations union and intersection, are also often iterated, but the iterations are not given separate names. In print, summation and product are represented by special symbols; but other iterated operators often are denoted by larger variants of the symbol for the ordinary binary operator. Thus, the

iterations of the four operations mentioned above. In general, there is more than one way to extend a binary operation to operate on finite sequences, depending on whether the operator is associative, and whether the operator has identity elements.¹²

Iterated binary operations are used to represent an operation that will be repeated over a set subject to some constraints. Typically the lower bound of a restriction is written under the symbol, and the upper bound over the symbol, though they may also be written as superscripts and subscripts in compact notation. Interpolation is performed over positive integers from the lower to upper bound, to produce the set which will be substituted into the index (below denoted as i) for the repeated operations. In mathematics, summation by parts transforms the summation of products of sequences into other summations, often simplifying the computation or (especially) estimation of certain types of sums. It is also called Abel's lemma or Abel transformation, named after Niels Henrik Abel who introduced it in 1826.^[1]

- It is used to prove Kronecker's lemma, which in turn, is used to prove a version of the strong law of large numbers under variance constraints.
- It may be used to prove Nicomachus's theorem that the sum of the first cubes equals the square of the sum of the first positive integers.^[2]
- Summation by parts is frequently used to prove Abel's theorem and Dirichlet's test.^{13,14}

Results

In mathematics, a product is the result of multiplication, or an expression that identifies objects (numbers or variables) to be multiplied, called factors. The order in which real or complex numbers are multiplied has no bearing on the product; this is known as the commutative law of multiplication.^{15,16} When matrices or members of various other associative algebras are multiplied, the product usually depends on the order of the factors. Matrix multiplication, for example, is non-commutative, and so is multiplication in other algebras in general as well.

There are many different kinds of products in mathematics: besides being able to multiply just numbers, polynomials or matrices, one can also define products on many different algebraic structures. In general, whenever one has two mathematical objects that can be combined in a way that behaves like a linear algebra tensor product, then this can be most generally understood as the internal product of a monoidal category. That is, the monoidal category captures precisely the meaning of a tensor product; it captures exactly the notion of why it is that tensor products behave the way they do. More precisely, a monoidal category is the class of all things (of a given type) that have a tensor product. The empty product on numbers and most algebraic structures has the value of 1 (the identity element of multiplication), just like the empty sum has the value of 0 (the identity element of addition). However, the concept of the empty product is more general, and requires special treatment in logic, set theory, computer programming and category theory.^{17,18}

All of the previous examples are special cases or examples of the general notion of a product. For the general treatment of the concept of a product, see product (category theory), which describes how to combine two objects of some kind to create an object, possibly of a different kind. But also, in category theory, one has:

- the fiber product or pullback,
- the product category, a category that is the product of categories.
- the ultraproduct, in model theory.
- the internal product of a monoidal category, which captures the essence of a tensor product.

A function's product integral (as a continuous equivalent to the product of a sequence or as the multiplicative version of the normal/standard/additive integral. The product integral is also known as "continuous product" or "multiplicational". Complex multiplication, a theory of elliptic curves.

The factorial operation is encountered in many areas of mathematics, notably in combinatorics, where its most basic

use counts the possible distinct sequences – the permutations – of distinct objects: there are $n!$ permutations of n distinct objects. In mathematical analysis, factorials are used in power series for the exponential function and other functions, and they also have applications in algebra, number theory, probability theory, and computer science.^{19,20}

Much of the mathematics of the factorial function was developed beginning in the late 18th and early 19th centuries. Stirling's approximation provides an accurate approximation to the factorial of large numbers, showing that it grows more quickly than exponential growth. Legendre's formula describes the exponents of the prime numbers in a prime factorization of the factorials, and can be used to count the trailing zeros of the factorials. Daniel Bernoulli and Leonhard Euler interpolated the factorial function to a continuous function of complex numbers, except at the negative integers, the (offset) gamma function.^{21,22}

Many other notable functions and number sequences are closely related to the factorials, including the binomial coefficients, double factorials, falling factorials, primorials, and subfactorials. Implementations of the factorial function are commonly used as an example of different computer programming styles, and are included in scientific calculators and scientific computing software libraries. Although directly computing large factorials using the product formula or recurrence is not efficient, faster algorithms are known, matching to within a constant factor the time for fast multiplication algorithms for numbers with the same number of digits.²³

III.CONCLUSIONS

- In mathematics, summation by parts transforms the summation of products of sequences into other summations, often simplifying the computation or (especially) estimation of certain types of sums. It is also called Abel's lemma or Abel transformation, named after Niels Henrik Abel who introduced it in 1826.^[1] It is used to prove Kronecker's lemma, which in turn, is used to prove a version of the strong law of large numbers under variance constraints.^{24,25}
- It may be used to prove Nicomachus's theorem that the sum of the first cubes equals the square of the sum of the first positive integers.^[2]
- Summation by parts is frequently used to prove Abel's theorem³² and Dirichlet's test.

A summation-by-parts (SBP) finite difference operator conventionally consists of a centered difference interior scheme and specific boundary stencils that mimics behaviors of the corresponding integration-by-parts formulation.^{[3][4]} The boundary conditions are usually imposed by the Simultaneous-Approximation-Term (SAT) technique.^[5] The combination of SBP-SAT^{30,31} is a powerful framework for boundary treatment. The method is preferred for well-proven stability for long-time simulation, and high order of accuracy.^{26,27}

In mathematics, especially the usage of linear algebra in mathematical physics, Einstein notation (also known as the Einstein summation convention or Einstein summation notation) is a notational convention that implies summation over a set of indexed terms in a formula, thus achieving brevity. As part of mathematics it is a notational^{28,29} subset of Ricci calculus; however, it is often used in physics applications that do not distinguish between tangent and cotangent spaces. It was introduced to physics by Albert Einstein in 1916.^[1]

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