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[ijirset@gmail.com](mailto:ijirset@gmail.com)

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# Simulation of Annealing for Portfolio Selection in Mean-Pseudo-Variance Fuzzy Model

Farbod Dehghani

Department of Economics, Western Michigan University, Kalamazoo MI 49008-5330 USA

## ABSTRACT

The issue of portfolio selection is one of the most important issues that investors face in order to establish a logical balance between return and risk in their investment. In this regard, the main goal pursued in this article is to provide an efficient method for portfolio selection. In this study, in order to formulate return and risk, the fuzzy model (mean-pseudo-variance) is presented in the framework of credit theory, and an algorithm is proposed to solve this non-linear model, in which annealing simulation is used to select the portfolio and simulation Fuzzy is used to calculate the mean and pseudo-variance of the fuzzy returns. Finally, to check the effectiveness of the proposed algorithm, several numerical examples were solved, and the results showed that the proposed algorithm, compared to other algorithms, provides better solutions in It offers a shorter time.

**KEYWORDS:** portfolio selection, mean-pseudo-variance model, fuzzy variable, fuzzy simulation, annealing simulation

## 1.INTRODUCTION

The first model proposed by Markowitz for portfolio selection is the mean-variance model, which uses the first two moments of the return distribution to find a logical balance between these two criteria in order to obtain the minimum risk for a certain level of expected return or maximum. The expected return is at a certain level of risk [1]. Actually, in this model, the variance of return distribution is used to measure risk. But this criterion also has limitations [2-3]. One of its limitations is that larger returns are considered as unfavorable as smaller returns than the average, because larger returns also lead to increased variance and thus increased risk. This criterion is also incomplete for stock returns with asymmetric possible distribution to measure risk, because based on the variance, it may lead to getting a smaller expected return for the portfolio by increasing the removal of returns that are larger and lower than the average. With the studies conducted on different markets, there are empirical evidences that show the absence of symmetry and also the normality of the distribution of stock returns [4-7], so another measure representing the risk in the portfolio should have been introduced. One of the criteria introduced by Markowitz is pseudo-variance, in which only the deviations of returns smaller than the average are used to measure risk [8]. The basic assumption in using the Markowitzian probabilistic classical model is that the market condition of a share in the future can be predicted only by using the data related to the past of that share. This is despite the fact that it is very difficult to accept such an assumption in today's variable markets. In addition, when there is no sufficient historical data related to a stock, it is not possible to use probabilistic models. To solve this problem, many researchers have suggested the use of fuzzy variables, so that while taking into account the past information of stocks, the professional judgments of experts can also be used [9-13]. In 2002, Liu defined the expected value and the variance of the fuzzy random variable, respectively, in order to measure the return and risk of a fuzzy portfolio [14]. After that, in the framework of the validity theory, several fuzzy models including mean-variance model [15], mean-pseudo-variance [16] and ... were introduced. | In this study, in order to select a portfolio based on the fuzzy mean-pseudo-variance model, a solution has been proposed in which an innovative method is combined with annealing simulation in order to achieve a portfolio close to the optimum and to calculate the mean and pseudo-variance of portfolio stock returns. Fuzzy simulation was used. The structure of the article is as follows: in part 2, a brief overview of fuzzy concepts is given, in part 3, the mean-pseudo-variance fuzzy model is explained, and in part 4, the proposed algorithm is introduced in detail, and in the final part, with some numerical examples, the efficiency The model is shown.



II. INTRODUCTION TO FUZZY CONCEPTS

The theory of fuzzy sets was formally proposed for the first time by Professor Lotfizadeh in 1965 [17]. One of the most important parts of the fuzzy theory is the theory of possibility [18], which is somewhat similar to the theory of probability with the difference that in the theory of possibility, the possibility of the occurrence of an event, in addition to statistical studies about it, is the possibility of the occurrence of that event. It is also logically dependent. Despite the importance and high utility of the probability of a fuzzy event, this factor lacks self-duality. Not having this property will lead to the maximum possibility of a fuzzy event (i.e. 1) not being able to guarantee the definite occurrence of this event, and as a result, this value cannot be trusted. In 2002, Liu and Liu presented the validity theory [14] and defined the validity of a fuzzy set as a competing option for the possibility of a fuzzy set. A distinct advantage of credit is its self-duality property. That is, when the validity value of an event is equal to one, that event will definitely happen. Therefore, the validity theory is based on the presentation The basic concepts proposed in [19] were rapidly expanded. It should be noted that in this article, whenever the validity of a fuzzy event is mentioned, it means the probability of occurrence of a fuzzy event. In order to understand as much as possible the content presented in this article, below is a brief overview of the required knowledge related to fuzzy variables and validity theory.

Definition 1: Suppose  $\xi$  is a fuzzy variable with membership function  $\mu, u, r$  are real numbers, the probability of a fuzzy event with characteristic  $\xi \geq 1$  is equal to equation 1.

$$\text{pos}\{\xi \geq r\} = \sup \mu(u) \quad , u \geq r$$

Definition 2: Suppose  $\xi$  is a fuzzy variable with membership function  $\mu, u, r$  are real numbers, the degree of requirement of a fuzzy event with  $\xi \geq 1$  characteristic is equal to equation 2

$$\text{Nec}\{\xi \geq r\} = 1 - \text{pos}\{\xi < r\} = 1 - \sup \mu(u) \quad , u < r$$

Definition 3: Suppose  $\xi$  is a fuzzy variable with membership function  $\mu, u, r$  are real numbers, the degree of validity of a fuzzy event with the characteristic  $\xi \geq 1$  is equal to the arithmetic mean of the probability and requirement value of that fuzzy event, that is:

$$\text{Cr}\{\xi \geq r\} = \frac{1}{2}(\text{pos}\{\xi \geq r\} + \text{Nec}\{\xi \geq r\})$$

Definition 4: If  $\xi$  is a fuzzy variable, then its expected value will be defined as follows:

$$E(\xi) = \int_0^{\infty} \text{Cr}\{\xi \geq r\} dr - \int_{-\infty}^0 \text{Cr}\{\xi \leq r\} dr$$

Definition 5: If  $\xi$  is a fuzzy variable with finite expected value  $e$ , then the variance of that variable will be defined as follows:

$$V[\xi] = E\left[(\xi - e)^2\right]$$

Definition 6: If  $\xi$  is a fuzzy variable with expected value range, then the pseudo variance of that variable will be defined as follows:

$$S[\xi] = E\left[\left((\xi - e)^-\right)^2\right]$$

III. MEAN-PSEUDO-VARIANCE FUZZY MODEL

If the pseudo-variance of the fuzzy variable is accepted as a risk measure for optimal portfolio selection in the fuzzy environment, then for the fuzzy modeling of the mean pseudo-variance for portfolio selection,  $\xi$  is considered as a fuzzy variable for returns. In each share,  $X_i$  is considered as the weight (investment ratio) of each share in the portfolio, and the return ( $\xi_i$ ) of each share is defined as follows:

$$\xi_i = \frac{p'_i + d_i - p_i}{p_i}$$

In the above relationship,  $p_i, p'_i, d_i$  are equal to the price of share  $i$  at the present time, the estimated price of share  $i$  in the next period and the amount of estimated income during the desired period. Since  $p_i$  and  $d_i$  are currently uncertain for the investor, if they are estimated as fuzzy variables,  $\xi_i$  is also a fuzzy variable. Therefore, the return of a  $n$  share portfolio with a weighted vector  $(x_1, x_2, \dots, x_n)$  equal to  $\xi = \xi_1 x_1 + \xi_2 x_2 + \dots + \xi_n x_n$  is also a fuzzy variable. The mean-pseudo-



variance fuzzy model, when the investor wants to maximize the expected return with the maximum risk with the value of  $L_a$ , is shown as follows:

$$\begin{aligned} \text{Maximize} \quad & E[\xi_1 x_1 + \xi_2 x_2 + \dots + \xi_n x_n] \\ \text{Subject to} \quad & S[\xi_1 x_1 + \xi_2 x_2 + \dots + \xi_n x_n] \leq \gamma \\ & x_1 + x_2 + \dots + x_n = 1 \\ & x_i \geq 0, \quad i = 1, 2, \dots, n \end{aligned}$$

The portfolio selection problem, if the investor wants to minimize the risk to obtain the expected return, at least as much as  $\alpha$ , is modeled as follows:

$$\begin{aligned} \text{Minimize} \quad & S[\xi_1 x_1 + \xi_2 x_2 + \dots + \xi_n x_n] \\ \text{Subject to} \quad & E[\xi_1 x_1 + \xi_2 x_2 + \dots + \xi_n x_n] \geq \alpha \\ & x_1 + x_2 + \dots + x_n = 1 \\ & x_i \geq 0, \quad i = 1, 2, \dots, n \end{aligned}$$

It should be noted that in the above models, if the stock returns are independent fuzzy variables with simple membership functions such as triangular or trapezoidal, the definitive model of each of them can be found and solved by analytical methods, but if the fuzzy variables have functions If the membership is more complex, this action will not be possible [16].

#### IV. ANNEALING SIMULATION ALGORITHM BASED ON FUZZY SIMULATION

In this part, in order to solve the fuzzy mean-pseudo-variance model with different membership functions for the fuzzy variables of stock returns, a combination algorithm of annealing simulation and fuzzy simulation is proposed, which will be examined in detail below.

##### 4.1. Fuzzy simulation

Fuzzy simulation was first introduced in 1998 by Liu and Elamora [20] and successfully used by Liu to solve fuzzy optimization problems [21]. On this, Liu proved the convergence of fuzzy simulation and showed its efficiency in approximating the exact size of functions [22]. In this article, fuzzy simulation is used to calculate the average and pseudo-variance of fuzzy returns. Suppose  $\xi_i$  fuzzy variable related to stocks  $i$  with membership function  $x_i, \mu_i$  the decision variable of the problem is equal to the weight of each of the stocks in the portfolio and  $n$  number of stocks under consideration. As explained earlier, in order to calculate the expected value of a fuzzy variable, the validity of that fuzzy variable should be calculated. Therefore, in order to calculate the mean and pseudo-variance of the portfolio return, the value  $Cr\{f(x, \xi) > r\}$  should be calculated, where  $f(x) = \xi_1 x_1 + \xi_2 x_2 + \dots + \xi_n x_n$ .

For this purpose, real random numbers must be generated  $w_{ij}$  with a condition  $\mu_i(w_{ij}) \geq \epsilon$ . If the number of simulation iterations is considered to be  $N$ , then  $w_{ij}$  means the random number generated for the share  $i$  in iteration  $j$  ( $i=1, 2, \dots, n$ ;  $j=1, 2, \dots, N$ ), where  $\epsilon$  is a very small positive real number and  $N$  is an integer of size It is big enough. After generating random values  $w_{ij}$ , the value  $Cr\{f(x, \xi) > r\}$  can be calculated through the following equation:

$$Cr\{f(x, \xi) \geq r\} = \frac{1}{2} \left( \max_{1 \leq i \leq N} \left\{ \min_{1 \leq j \leq n} \mu_j(w_{ji}) \mid f(x, \xi) \geq r \right\} + 1 - \max_{1 \leq j \leq n} \left\{ \min_{1 \leq i \leq N} \mu_j(w_{ji}) \mid f(x, \xi) < r \right\} \right)$$

The fuzzy simulation process to calculate  $E = (\xi_1 x_1 + \xi_2 x_2 + \dots + \xi_n x_n)$  is summarized as follows:

step1: First, set  $E$  (expected value of the portfolio) to zero and go to the next step ( $E=0$ )

Step 2: Generate random  $W_{ij}$  numbers for ( $i=1, 2, \dots, n$ ;  $j=1, 2, \dots, N$ ) and with  $\mu_i(w_{ij}) \geq \epsilon$  condition.

Step3: Calculate the values of  $a$  and  $b$ , which are the minimum and maximum returns of the portfolio, respectively, in  $N$  times of testing. (The meaning of  $W_i$  is the  $W_i = (W_{1j}, W_{2j}, \dots, W_{nj})$   $n$  vector)

$$a = \min_{1 \leq i \leq N} \{f(x, w_i)\} \quad \& \quad b = \max_{1 \leq i \leq N} \{f(x, w_i)\}$$

Step 4: Generate the number  $r$  randomly in the interval  $(a, b)$

Step5: if  $r \geq 0$ , then  $E + Cr\{f(x, \xi) \geq r\} \quad \mathbf{E} \rightarrow$

Step6: if  $r < 0$ , then  $E - Cr\{f(x, \xi) \geq r\} \quad \mathbf{E} \rightarrow$

Step7: Do the fourth to fifth steps  $N$  times and then go to the next step.

Step8:  $E[f(x, \xi)] = a \nu_0 + b \lambda_0 + E \cdot (b - a) / N$

$Q$  from obtaining the expected value of the portfolio ( $E$ ), in order to calculate the pseudo-variance

The portfolio is only enough that in the above algorithm  $f(x, \xi)$  by  $g(x, \xi)$  to be replaced. The value of  $g(x, \xi)$  can be calculated from the following equation.

$$g(x_i, \xi_i) = \begin{cases} (f(x_i, \xi_i) - E)^2 & \text{if } (f(x_i, \xi_i) - E) < 0 \\ 0 & \text{if } (f(x_i, \xi_i) - E) \geq 0 \end{cases}$$

#### 4.2. Simulation of annealing

Annealing simulation is a meta-heuristic local search method based on the Monte Carlo model, which is based on the relationship between the atomic structure, entropy and temperature during the cooling of a material and is similar to the physical phenomenon of annealing or annealing of solid materials, widely used in the field. Various combinatorial optimization problems are used. This method was first presented by Kirkpatrick et al. in 1983 to solve optimization problems [23]. In simulation, annealing works by searching in sets of possible solutions to find the optimal solution. In the process of this search, the algorithm moves from an initial solution (1), which is chosen randomly or based on the available information of the problem, to a solution in its neighborhood (j) and in addition to accepting a solution that improves the objective function Unfavorable answers are also accepted on the condition that the following condition, which is called the Metropolis criterion, is satisfied:

$$e^{\pm\left(\frac{\Delta f}{T}\right)} \geq \text{Random}[0,1]$$

In the above relation,  $T$  is the current temperature and  $\Delta f$  is the amount of change in the objective function. SA repeats this process  $L_n$  (the length of the Markov chain) at each level of temperature and the parameter  $T_r$  is slowly reduced using the temperature reduction function until the stopping condition is met. In this article, annealing simulation was used to select the portfolio based on the mean-pseudo-variance fuzzy model, and the implementation steps of this algorithm are explained below.

##### 4.2.1.SA algorithm components and parameters

###### 4.2.1.1. Display the structure of the answer

Representation of the structure of the solution The structure of the solution shows a point of the feasible space of the problem, in this article each solution  $X = (x_1, x_2, \dots, x_n)$  is represented by the line vector  $[X_1 X_2, \dots, X_n]$  The mechanism of neighborhood solution selection in this algorithm is such that the limit  $X_1 + X_2 + \dots + X_n = 1$  will always be maintained.

###### 4.2.1.2. Choosing the initial answer

The following algorithm is used to select an initial feasible solution:

Step 1: Generate the vector  $C = (c_1, c_2, \dots, c_n)$  where all its domains are random numbers in the interval  $[0,1]$

Step 2: Calculate the  $X$  vector using the  $C$  vector and through the following equation.

$$x_i = \frac{c_i}{\sum_{i=1}^n c_i} \quad i = 1, 2, 3, \dots, n$$

Step 3: With the help of fuzzy simulation, check the feasibility of solution  $X$ , if the solution is not feasible, return to the first step, and otherwise, accept the current solution as the initial solution.

###### 4.2.1.3. The mechanism of creating neighborhood answer

In order to create a neighborhood solution, a method is presented in reference [24]. But since the vector  $X$  has infinite neighborhood solutions in every iteration and only a small part of them leads us to the optimal solution, therefore, in this article, an innovative method was used in order to obtain a suitable neighborhood solution. More details of this method are given in the following algorithm: Step 1: Consider the current answer  $X_c$ .

Step 2: Choose two numbers of  $X_c$  vector vectors at random and consider them,  $i$  and  $j$ .

Step 3: Generate random number  $r_1$  between zero and  $X_i$  and random number  $r_2$  between zero and  $X_j$ .

Step 4: Subtract the random number  $r_1$  from  $X_i$  and consider the resulting vector  $X_1$ .

Step 5: Subtract the random number  $r_2$  from  $X_j$  and consider the resulting vector as  $X_2$ .

Step 6 Calculate the mathematical expectation and pseudo-variance of  $X_1$  and  $X_2$  answers using fuzzy simulation and choose the feasible solution with the highest mathematical expectation and consider it  $X_3$ . If none of them are feasible, go to step 8.

Step 7: If  $X_3$  has a higher mathematical expectation than  $X_n$ , replace it with  $X_n$ . (If  $X_n$  has no value, immediately replace it with  $X_3$ ) and then go to the next step.

Step 8: Do steps 1 to 7  $m$  times and then go to the next step.

Step 9: Consider  $X_n$  as the neighborhood solution of  $X_c$ .

###### 4.2.1.4. Selecting the initial temperature

The selection of the initial temperature has a great effect on the final solution of the problem, if the initial temperature is low, the probability of accepting worse solutions is reduced and the system may remain in the local optimal solution. If we want the final solution to be independent from the starting solution, the initial temperature should be considered high enough, so that it is possible to exchange the neighboring solutions almost freely, otherwise the final solution will always be close to the initial solution. White proposed the theory of  $T_0$  being equivalent to the standard deviation of system costs from the average cost [25]. In this method, the initial temperature is considered equal to the standard deviation of the value of the objective

function in a certain number of times of running the program in an unstable state, and at each stage, it is calculated based on 5 samples of the initial temperature (relationship below).

$$T_0 = \sqrt{\frac{\sum_{j=1}^N (Obj(j) - Mean)^2}{N-1}}, \quad Obj(j) = \text{The objective function of the problem}$$

4.2.1.5. Temperature reduction mechanism

The process of lowering the temperature and moving towards cooling the system requires the following rule:

$$T_{r+1} = \gamma T_r$$

$\gamma$  It is the coefficient of temperature reduction, which is constant less than 1 and usually between 0.5 and 0.99. It is considered and this value depends on the problem under consideration. In the provided software (MATLAB), it is possible to receive the  $\gamma$  value as an input.

4.2.1.6. Markov chain length

Another important parameter in the qualitative and more accurate determination of the solution of the portfolio model with the SA algorithm is the number of points searched in the solution space of the problem at a constant temperature to ensure a search close to all possible solutions in the algorithm. The easiest suggestion to determine the value of  $L_n$  is to choose a value that is based on the size of the problem. Since the value of this parameter also depends on the investigated problem, the value of  $L_n$  will also be asked from the user as input values like the parameter  $\gamma$ .

4.2.2. Acceptance mechanism of nominated answers

Another component that determines the quality and speed of reaching the answer is the way of accepting or rejecting and searching again in the answer space. The mechanism of accepting the answers is according to the following algorithm:

Step 1: Calculate the value  $d = E(X_n) - E(X_c)$

Step 2: If  $d \geq 0$ , then set  $X_c$  and  $X_b$  (the best answer to  $X_n$ ).

Step 3: If  $d \leq 0$ , generate a random number  $r$  in the interval  $[0,1]$  and if  $r \leq e^{(-d/T)}$ , then set  $X_c$  equal to  $X_n$ .

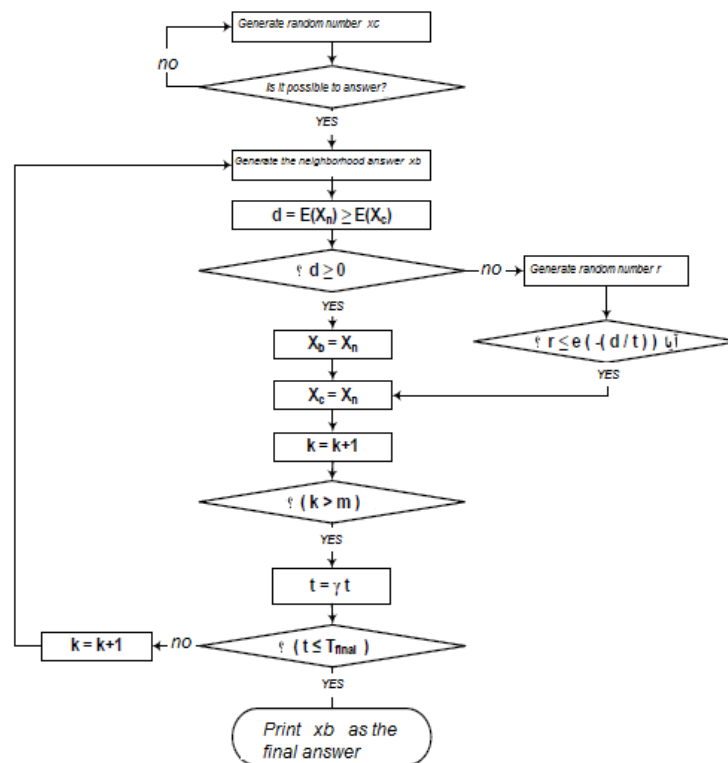


Figure 1: Diagram of the proposed algorithm



V. SAMPLE EXAMPLE

In this part, in order to evaluate the effectiveness of the proposed algorithm in portfolio selection, some numerical examples have been investigated. All the calculations related to these examples are done with MATLAB software. The parameters of the model in these examples are as follows: 3000 repetitions of fuzzy simulation, length of the Markov chain, 30, parameter m in creating the neighborhood solution, 20, initial temperature, 100 degrees, final temperature 0.0001 degrees and cooling rate 0.9

10 shares with fuzzy returns are considered according to Table 1 [16], the return rate of the first 7 shares has a triangular membership function, the next two shares have a bell-shaped membership function, and finally the return of the tenth stock has a normal membership function.

Table 1: Fuzzy returns of 10 stocks

take stock	Fuzzy efficiency			take stock	Fuzzy efficiency		
1.	-0.3	1.8	2.3	6.	-0.8	2.1	3.2
2.	-0.4	2	2.2	7.	-0.7	2.15	3.4
3.	-0.5	1.9	2.7	8.	-1	2.2	3.6
4.	-0.6	2	2.8	9.	-1.1	2.25	3.8
5.	-0.7	2.05	2.7	10.	-1.2	2.3	4

Example 1: Suppose the maximum allowed risk is 0.6 be In this case, the fuzzy model of this problem to maximize the expected return of the portfolio will be as follows.

$$\begin{aligned}
 &\text{Maximize} && E[\xi_1x_1 + \xi_2x_2 + \dots + \xi_{10}x_{10}] \\
 &\text{Subject to} && S[\xi_1x_1 + \xi_2x_2 + \dots + \xi_{10}x_{10}] \leq 0.6 \\
 &&& x_1 + x_2 + \dots + x_{10} = 1 \\
 &&& x_i \geq 0, \quad i = 1, 2, \dots, 10
 \end{aligned}$$

This problem has been solved in reference [16] with the help of a hybrid algorithm consisting of genetics (GA) and fuzzy simulation and has resulted in a solution of 1/60. This is despite the fact that the algorithm proposed in this article was able to significantly improve the solution of the problem in a period of 801 seconds and achieve an expected efficiency of 1.7332 for the solution presented in Table 2. The convergence diagram of the objective function is shown in Figure 2.

Table 2: Investment ratios in 10 stocks

Method	1	2	3	4	5	6	7	8	9	10	Expected return
SA	0	0	0	0.115	0.113	0.405	0.001	0.366	0	0	1.7332
GA	0.208	0.111	0.191	0.0692	0.0964	0.324	0	0	0	0	1.6

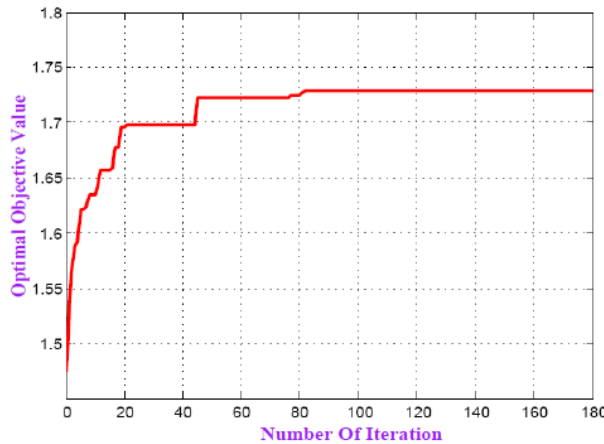


Figure 2: Convergence of the objective function in example 1

In order to test the proposed algorithm with respect to changing the parameters, a numerical example was solved with different parameters. The results are given in Table 3, in order to compare the results, the percentage difference of the obtained value with the best (maximum) value obtained from different parameters was used as relative error. The obtained results indicate that the highest error value is equal to 0.9 and less than 1%, so the presented combined algorithm is resistant to changing the parameters.

Table 3: Values of portfolio expected returns with different parameters

Relative error(%)	Expected return	M	$L_n$	$\square$	$T_{final}$	$T_0$	number
0.26	1.73	20	30	0.9	0.001	100	1.
0.54	1.73	20	40	0.9	0.001	100	2.
0.06	1.73	25	45	0.9	0.001	100	3.
0.80	1.73	20	30	0.9	0.001	100	4.
0	1.72	20	20	0.9	0.001	100	5.
0.73	1.73	40	45	0.9	0.001	100	6.
0.42	1.73	30	45	0.9	0.001	100	7.
0.04	1.73	35	45	0.9	0.001	100	8.
0.38	1.72	20	40	0.9	0.001	100	9.
0.74	1.73	20	20	0.9	0.001	100	10.

Example 2: Consider the data in Table 1 again and this time assume that the investor intends to minimize the investment risk for a minimum expected return of 1.5. This problem is modeled as follows:

$$\begin{aligned} &\text{Maximize} && S[\xi_1x_1 + \xi_2x_2 + \dots + \xi_{10}x_{10}] \\ &\text{Subject to} && E[\xi_1x_1 + \xi_2x_2 + \dots + \xi_{10}x_{10}] \geq 1.5 \\ &&& x_1 + x_2 + \dots + x_{10} = 1 \\ &&& x_i \geq 0, \quad i = 1, 2, \dots, 10 \end{aligned}$$

This problem was also solved in reference [16] with the help of genetic algorithm and led to achieving the minimum risk of 0.32. But the algorithm proposed in this article was able to achieve the minimum risk of 0.127 for the answer given in Table 3.





Table 2: Investment ratios in 10 stocks

0.32	0.76	Expected return
GA	SA	Method
0.473	0.021	1.
0.076	0.198	2.
0.666	0.931	3.
0.542	0.617	4.
0.483	0.380	5.
0.211	0.191	6.
0.314	0.862	7.
0.140	0.056	8.
0.231	0.426	9.
0.781	0.443	10.

## VI. CONCLUSION

In this study, the mean-pseudo-variance fuzzy model was used to select the portfolio, in which the returns were assumed to be fuzzy. To solve this model, a hybrid algorithm consisting of annealing simulation and fuzzy simulation was proposed. The presented numerical examples and its comparison with the results obtained from the genetic algorithm showed that the presented algorithm was able to provide far better and more accurate results. In addition, although the execution time of the algorithm is not mentioned in the reviewed articles, in any case, due to the population-based nature of the genetic algorithm used in these articles, it can be concluded that the algorithm proposed in this article has a much shorter execution time. compared to other similar algorithms. Also, this algorithm is resistant to changing parameters, which can indicate the efficiency of this solution.

## VII. ABBREVIATIONS

Not applicable

## VIII. DECLARATIONS

- Availability of data and materials
- The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.
- Competing interests  
The author declare that he has no competing interests
- Funding  
None.
- Acknowledgements  
Not applicable
- Authors contribution  
This paper has only one author and he is responsible for all the materials provided in this paper including but not limited to: research design, data analysis, data interpretation, writing the paper and approving final manuscript.

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