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# An Application of Fractional Intergral Operator Involving Extended Mittag-Leffler Function as Its Kernel

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**ABSTRACT:** This paper is devoted to the study of fractional calculus with an integral and differential operators containing the following family of extended Mittag–Leffler function:

$$E_{\alpha,\beta}^{\gamma,\delta;c,d}(z; p, q) = \sum_{n=0}^{\infty} \frac{B_p(\gamma+n,c-\gamma) B_q(\delta+n,d-\delta) (c)_{np} (d)_{nq}}{B_p(\gamma,c-\gamma) B_q(\delta,d-\delta) \Gamma(\alpha n + \beta) n!} z^n$$

in its kernel. Also, we further introduce a certain number of consequences of fractional integral and differential operators containing the said function in their kernels.

**KEYWORDS:** Fractional integral operator , Fractional differential operator , Mittag–Leffler function , Lebesgue measurable function, Extended Mittag–Leffler function.

## I. INTRODUCTION

The Mittag–Leffler functions appear in special functions as a solution of fractional order integral and differential equations. Some interesting applications of the Mittag Leffler function are considered as follows: studied of kinetic equation, the telegraph equations [5], random walks, Levy flights, supper diffuse transport and complex system.

We begin with the Mittag–Leffler functions  $E_{\alpha}(z)$  and  $E_{\alpha,\beta}(z)$  are defined by the following series:

$$E_{\alpha}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)} \quad z \in \mathbb{C}; \Re(\alpha) > 0 \tag{1.1.1}$$

and

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)} \quad z, \beta \in \mathbb{C}; \Re(\alpha) > 0, \tag{1.1.2}$$

respectively. For further study of Mittag–Leffler function such as generalizations and applications , the readers may refer to the current work of researchers (for example) Džrbašjan [7], Kilbas and Saigo [15], Gorenflo and Mainardi [8] , Gorenflo et al. ([9,11]), Kilbas et al. ([16], and Saigo and Kilbas [22]. In recent years, the Mittag–Leffler function (1.1.1) and some of its variety of generalizations have been numerically investigated in the complex plane (see [14,24]). A generalization of the Mittag–Leffler function  $E_{\alpha,\beta}(z)$  of (1.1.2) was introduced by Prabhakar [20] as follows:

$$E_{\alpha,\beta}^{\gamma}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n z^n}{\Gamma(\alpha n + \beta) n!} \quad z, \beta \in \mathbb{C}; \Re(\alpha) > 0, \tag{1.1.3}$$

where  $(\gamma)_n$  denote the well-known Pochhammer Symbol which is defined by

$$(\gamma)_n = \begin{cases} 1, & \text{when } (n = 0, \gamma \neq 0) \\ \gamma(\gamma + 1) \dots (\gamma + n - 1), & \text{when } (n \in \mathbb{N}, \gamma \in \mathbb{C}) \end{cases} .$$

Obviously, the following special cases are satisfied:

$$E_{\alpha,\beta}^1(z) = E_{\alpha,\beta}(z) = E_{\alpha,1}^1(z) = E_{\alpha}(z). \tag{1.1.4}$$

Diaz and Pariguan [see nisar (k, s) fractional calculus] have introduced the Pochhammer  $p$  –symbols and  $p$  –Gamma function defined as follows

$$(\gamma)_{np} = \begin{cases} 1, & \text{when } (n = 0, \gamma \neq 0) \\ \gamma(\gamma + p) \dots (\gamma + (n - 1)p), & \text{when } (n \in \mathbb{N}, \gamma \in \mathbb{C}, p > 0) \end{cases}$$

And

$$\Gamma_p(\gamma) = \int_0^\infty t^{\gamma-1} e^{-\frac{t^p}{p}} dt$$

In recent times many researchers have investigated the importance and great consideration of Mittag-Leffler function in the theory of special functions for exploring the generalization and some applications. Many extensions for these functions are found in [1-4,10]. Srivastava and Tomovski [25] have defined further generalization of the Mittag-Leffler function  $E_{\alpha,\beta}^{\gamma,\kappa}(z)$  of (3), which is defined as follows:

$$E_{\alpha,\beta}^{\gamma,\kappa}(z) = \sum_{n=0}^\infty \frac{(\gamma)_{n\kappa}}{\Gamma(\alpha n + \beta)} \frac{z^n}{n!}, \tag{1.1.5}$$

where  $z, \beta, \gamma \in \mathbb{C}; \Re(\alpha) > \max\{0, \Re(\kappa) - 1\}; \Re(\kappa) > 0$ .

In the same paper, they have used the well-known right-sided Riemann-Liouville fractional integral, derivative and generalized Riemann-Liouville derivative operators (see [12,13,16,23]). Very recently Özarıslan and Yılmaz [19] have investigated an extended Mittag-Leffler function  $E_{\alpha,\beta}^{\gamma,c}(z; p)$ , which is defined as follows:

$$E_{\alpha,\beta}^{\gamma,c}(z; p) = \sum_{n=0}^\infty \frac{B_p(\gamma + n, c - \gamma)}{B_p(\gamma, c - \gamma)} \frac{(c)_n}{\Gamma(\alpha n + \beta)} \frac{z^n}{n!}, \tag{1.1.6}$$

where  $p \geq 0, \Re(c) > \Re(\gamma) > 0$  and  $B_p(x, y)$  is extended beta function defined in [6] as follows:

$$B_p(x, y) = \int_0^1 t^{x-1} (1 - t)^{y-1} e^{-\frac{t^p}{p}(1-t)} dt \tag{1.1.7}$$

for  $\Re(p) > 0, \Re(x) > 0$  and  $\Re(y) > 0$ . In the same paper, they defined the following result for the extended Mittag-Leffler function (1.1.6) as follows:

$$\left(\frac{d}{dz}\right)^n (z^{\lambda-1} E_{\alpha,\beta}^{\gamma,c}(\omega z^\alpha; p)) = z^{\lambda-n-1} E_{\alpha,\beta-n}^{\gamma,c}(\omega z^\alpha; p). \tag{1.1.8}$$

In this continuation of the study on the significance of fractional calculus, we start with the following preliminaries:

In this paper we investigated an extended Mittag-Leffler function  $E_{\alpha,\beta}^{\gamma,\delta;c,d}(z; p, q)$ , which is defined as follows:

$$E_{\alpha,\beta}^{\gamma,\delta;c,d}(z; p, q) = \sum_{n=0}^\infty \frac{B_p(\gamma + n, c - \gamma)}{B_p(\gamma, c - \gamma)} \frac{B_q(\delta + n, d - \delta)}{B_q(\delta, d - \delta)} \frac{(c)_{np} (d)_{nq}}{\Gamma(\alpha n + \beta)} \frac{z^n}{n!}, \tag{1.1.9}$$

where  $p, q \geq 0, \Re(c) > \Re(\gamma) > 0, \Re(d) > \Re(\delta) > 0$  and  $B_p(x, y)$  and  $B_q(x, y)$  are extended beta function defined as follows,

$$B_p(x, y) = \int_0^1 t^{x-1} (1 - t)^{y-1} e^{-\frac{t^p}{p}(1-t)} dt$$

And

$$B_q(x, y) = \int_0^1 t^{x-1} (1 - t)^{y-1} e^{-\frac{t^q}{q}(1-t)} dt$$

For  $\Re(p) > 0, \Re(x) > 0$  and  $\Re(y) > 0$ .

**Remark 1.1.1:** It should be noted that,

$$E_{\alpha,\beta}^{1,1;c,d}(z, 1, 1) = E_{\alpha,\beta}^{1,1}(z) = E_{\alpha,\beta}^{1,c}(z; 1) = E_{\alpha,1}^1(z) = E_{\alpha,1} = E_{\alpha}(z)$$

$$(1.1.9) \Rightarrow (1.1.6) \Rightarrow (1.1.3) \Rightarrow (1.1.2) \Rightarrow (1.1.1.)$$

Or

$$(1.1.9) \Rightarrow (1.1.5) \Rightarrow (1.1.3) \Rightarrow (1.1.2) \Rightarrow (1.1.1)$$

$\mathcal{L}(a, b)$  is the space of Lebesgue measurable of real or complex valued function if it is defined as

$$\mathcal{L}(a, b) = \left\{ f : \| f \|_1 = \int_a^b f(x) dx < \infty \right\}. \tag{1.1.10}$$

The left- and right-sided Riemann–Liouville fractional integral operators  $I_{a+}^\lambda$  and  $I_{b-}^\lambda$  are, respectively, defined as (see, e.g., [16]) follows:

$$(I_{a+}^\lambda f)(x) = \frac{1}{\Gamma(\lambda)} \int_a^x \frac{f(\tau)}{(\tau-x)^{1-\lambda}} d\tau, (x > a), \tag{1.1.11}$$

and

$$(I_{b-}^\lambda f)(x) = \frac{1}{\Gamma(\lambda)} \int_x^b \frac{f(\tau)}{(\tau-x)^{1-\lambda}} d\tau, (x < b), \tag{1.1.12}$$

where  $f(x) \in \mathcal{L}(a, b), \lambda \in \mathbb{C}$  and  $\Re(\lambda) > 0$ .

Similarly, for  $f(x) \in \mathcal{L}(a, b), \lambda \in \mathbb{C}, \Re(\lambda) > 0$  and  $n = [R(\lambda)] + 1$ , the left- and right-sided Riemann–Liouville fractional differential are defined as (see, e.g., [16]) follows:

$$(D_{a+}^\lambda f)(x) = \left(\frac{d}{dx}\right)^n (I_{a+}^{n-\lambda} f)(x), \tag{1.1.13}$$

$$(D_{a-}^\lambda f)(x) = \left(-\frac{d}{dx}\right)^n (I_{a-}^{n-\lambda} f)(x), \tag{1.1.14}$$

respectively.

A generalized form of fractional differential operator  $D_{a+}^\lambda$  (1.1.13) has been made by investigating the fractional differential operator  $D_{a+}^{\lambda, \nu}$  of order  $0 < \lambda < 1$  and type  $0 < \nu < 1$  with respect to  $x$  as follows:

$$(D_{a+}^{\lambda, \nu} f)(x) = \left( I_{a+}^{\nu(1-\lambda)} \frac{d}{dx} (I_{a+}^{(1-\nu)(1-\lambda)} f) \right) (x) \tag{1.1.15}$$

(see [23], [16], [21]). Obviously, when  $\nu = 0$  then (1.1.15) reduces to the operator  $D_{a+}^\lambda$  defined in (1.1.12).

We consider the following basic results for our study:

**Theorem 1.1.2** (Mathai and Haubold [17]) If  $\lambda, \mu \in \mathbb{C}, \Re(\lambda) > 0, \Re(\mu) > 0$ , then

$$I_{a+}^\lambda (\tau - a)^{\mu-1} = \frac{\Gamma(\mu)}{\Gamma(\lambda + \mu)} (x - a)^{\lambda + \mu - 1}. \tag{1.1.16}$$

**Theorem 1.1.3** (Srivatava and Manocha [26]) If a function  $f(z)$  is analytic and has a power series expansion such that  $f(z) = \sum_{n=0}^\infty a_n z^n$  in the disc  $|z| < \Re$ , then

$$D_{0,z}^\lambda \{z^{\mu-1} f(z)\} = \frac{\Gamma(\mu)}{\Gamma(\lambda + \mu)} \sum_{n=0}^\infty \frac{a_n (\mu)_n}{(\lambda + \mu)_n} z^n. \tag{1.1.17}$$

**Lemma 1.1.4** (Srivastava and Tomovski [25]) The following result holds true for fractional derivative operator  $D_{\alpha+}^{\mu, \nu} f$  as follows:

$$D_{\alpha+}^{\lambda, \nu} [(\tau - a)^{\beta-1}] (x) = \frac{\Gamma(\beta)}{\Gamma(\beta - \lambda)} (x - a)^{\beta - \lambda - 1}, \tag{1.1.18}$$

where  $x > a, 0 < \lambda < 1, 0 \leq \nu \leq 1, \Re(\beta) > 0$  and  $\Re(\lambda) > 0$ .

## II. AN INTEGRAL AND DIFFERENTIAL OPERATORS INVOLVING THE EXTENDED MITTAG–LEFFLER FUNCTION

In this section, we introduce fractional integrals and differentials of the extended Mittag–Leffler functions  $E_{\alpha, \beta}^{\gamma, \delta; c, d}(z; p, q)$ . In this continuation of the study of fractional calculus, we define the following integral operator as follows:

**Definition 2.1.1** If  $\gamma, \omega \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0$ , then

$$\left( \varepsilon_{\alpha+; \alpha, \beta}^{\omega; \gamma, \delta; c, d} f \right) (x) = \int_a^x (x - \tau)^{\beta-1} E_{\alpha, \beta}^{\gamma, \delta; c, d}(\omega(x - \tau)^\alpha; p, q) f(\tau) d\tau, \tag{1.1.19}$$

where  $x > \alpha$ . Substituting  $p = q = 0$ , then (1.1.19) reduces to the operator

$$\left( \varepsilon_{\alpha+; \alpha, \beta}^{\omega; \gamma, \delta} f \right) (x) = \int_a^x (x - \tau)^{\beta-1} E_{\alpha, \beta}^{\gamma, \delta}(\omega(x - \tau)^\alpha) f(\tau) d\tau, \tag{1.1.20}$$

see [25]. In fact, when  $\omega = 0$  then the integral operator in (1.1.20) reduces to the well known Riemann–Liouville fractional integral operator  $I_{a+}^\lambda$  defined in (1.1.11).

**Theorem 2.2.2** Suppose  $x > a (a \in \mathcal{R}_+ = [0, \infty)), \gamma, \delta, \lambda, \beta, \omega \in \mathbb{C}, \Re(\beta) > 0, \Re(\lambda) > 0$ ; then

$$I_{a+}^{\lambda} \left[ (\tau - a)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d} \left( \omega(\tau - a)^{\alpha}; p, q \right) \right] (x) = (x - a)^{\lambda+\beta-1} E_{\alpha,\beta+\lambda}^{\gamma,\delta;c,d} \left( \omega(x - a)^{\alpha}; p, q \right) \tag{1.1.21}$$

$$D_{a+}^{\lambda} \left[ (\tau - a)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d} \left( \omega(\tau - a)^{\alpha}; p, q \right) \right] = (x - a)^{\beta-\lambda-1} E_{\alpha,\beta-\lambda}^{\gamma,\delta;c,d} \left( \omega(x - a)^{\alpha}; p, q \right) \tag{1.1.22}$$

and

$$D_{a+}^{\lambda,\nu} \left[ (\tau - a)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d} \left( \omega(\tau - a)^{\alpha}; p, q \right) \right] (x) = (x - a)^{\beta-\lambda-1} E_{\alpha,\beta-\lambda}^{\gamma,\delta;c,d} \left( \omega(x - a)^{\alpha}; p, q \right). \tag{1.1.23}$$

**Proof**

$$\begin{aligned} I_{a+}^{\lambda} \left[ (\tau - a)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d} \left( \omega(\tau - a)^{\alpha}; p, q \right) \right] &= \frac{1}{\Gamma(\lambda)} \int_a^x \frac{(\tau - a)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d} \left( \omega(\tau - a)^{\alpha}; p, q \right)}{(x - \tau)^{1-\lambda}} d\tau \\ &= \frac{1}{\Gamma(\lambda) B_p(\gamma, c - \gamma) B_q(\delta, d - \delta)} \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) (c)_{np} (d)_{nq} \omega^n}{\Gamma(\alpha n + \beta) n!} \\ &\quad \times \int_0^x (\tau - a)^{\beta+\alpha n-1} (x - \tau)^{\lambda-1} d\tau \\ &= \frac{1}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta)} \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) (c)_{np} (d)_{nq} \omega^n}{\Gamma(\alpha n + \beta) n!} \\ &\quad \times \left( I_{a+}^{\lambda} \left[ (\tau - a)^{\beta+\alpha n-1} \right] \right). \end{aligned}$$

By the use of (1.1.16), we have

$$\begin{aligned} I_{a+}^{\lambda} \left[ (\tau - a)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d} \left( \omega(\tau - a)^{\alpha}; p, q \right) \right] &= \frac{1}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta)} \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) (c)_{np} (d)_{nq} \omega^n}{\Gamma(\alpha n + \beta) n!} (x - a)^{\beta+\lambda+\alpha n-1} \frac{\Gamma(\alpha n + \beta)}{\Gamma(\alpha n + \beta + \lambda)} \\ &= (x - a)^{\beta+\lambda-1} \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) (c)_{np} (d)_{nq} [\omega(x - a)^{\alpha}]^n}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta) \Gamma(\alpha n + \beta + \lambda) n!} \\ &= (x - a)^{\beta+\lambda-1} E_{\alpha,\beta+\lambda}^{\gamma,\delta;c,d} \left( \omega(x - a)^{\alpha}; p, q \right). \end{aligned}$$

This completes the proof of (1.1.21).

Now, we have

$$D_{a+}^{\lambda} \left[ (\tau - a)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d} \left( \omega(\tau - a)^{\alpha}; p, q \right) \right] = \left( \frac{d}{dx} \right)^{\lambda} \left\{ I_{a+}^{\beta-\lambda} \left[ (\tau - a)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d} \left( \omega(\tau - a)^{\alpha}; p, q \right) \right] \right\}$$

and using (1.1.21) this takes the following form:

$$D_{a+}^{\lambda} \left[ (\tau - a)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d} \left( \omega(\tau - a)^{\alpha}; p, q \right) \right] = \left( \frac{d}{dx} \right)^{\lambda} \left\{ (x - a)^{\beta-\lambda+n-1} E_{\alpha,\beta-\lambda+n}^{\gamma,\delta;c,d} \left( \omega(x - a)^{\alpha}; p, q \right) \right\}.$$

Applying (1.1.9), we have

$$D_{a+}^{\lambda} \left[ (\tau - a)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d} \left( \omega(\tau - a)^{\alpha}; c, d \right) \right] (x) = (x - a)^{\beta-\lambda-1} E_{\alpha,\beta-\lambda}^{\gamma,\delta;c,d} \left( \omega(x - a)^{\alpha}; p, q \right).$$

This completes the desired proof.

To prove (1.1.23), we have

$$\begin{aligned} \left( D_{a+}^{\lambda,\nu} \left[ (\tau - a)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d} \left( \omega(\tau - a)^{\alpha}; p, q \right) \right] \right) (x) &= D_{a+}^{\lambda,\nu} \left( \left[ \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) (c)_{np} (d)_{nq} \omega^n}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta) \Gamma(\alpha n + \beta) n!} (\tau - a)^{\alpha n + \beta - 1} \right] \right) (x) \\ &= \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) (c)_{np} (d)_{nq} \omega^n}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta) \Gamma(\alpha n + \beta) n!} \left( D_{a+}^{\lambda,\nu} \left[ (\tau - a)^{\alpha n + \beta - 1} \right] \right) (x). \end{aligned}$$

By applying (1.1.18), we get

$$\begin{aligned} \left( D_{a+}^{\lambda,\nu} \left[ (\tau - a)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d} \left( \omega(\tau - a)^{\alpha}; p, q \right) \right] \right) (x) &= \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) (c)_{np} (d)_{nq} \omega^n}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta) \Gamma(\alpha n + \beta) n!} \cdot \frac{\Gamma(\alpha n + \beta)}{\Gamma(\alpha n + \beta - \lambda)} (x - a)^{\alpha n + \beta - \lambda - 1} \\ &= (x - a)^{\beta-\lambda-1} \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) (c)_{np} (d)_{nq} [\omega(x - a)^{\alpha}]^n}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta) \Gamma(\alpha n + \beta - \lambda) n!} \\ &= (x - a)^{\beta-\lambda-1} E_{\alpha,\beta-\lambda}^{\gamma,\delta;c,d} \left( \omega(x - a)^{\alpha}; p, q \right), \end{aligned}$$

which completes the required proof.

1. Some properties of the operator  $(\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} f)(x)$

**Theorem 2.3.1** If  $\gamma, \omega \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\lambda) > 0$  and  $\Re(\beta) > 0$ , then

$$(\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} [(\tau - a)^{\mu-1}]) (x) = (x - a)^{(\mu+\beta-1)} \Gamma(\mu) E_{\alpha,\beta+\mu}^{\gamma,\delta;c,d}(\omega(x - a)^\mu; p, q) f(\tau) d\tau. \tag{1.1.24}$$

**Proof** From (1.1.19)

$$(\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} f)(x) = \int_a^x (x - \tau)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(x - \tau)^\alpha; p, q) f(\tau) d\tau.$$

Therefore, we have

$$\begin{aligned} (\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} [(\tau - a)^{\mu-1}]) (x) &= \int_a^x (x - \tau)^{\beta-1} (\tau - a)^{\mu-1} E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(x - \tau)^\alpha; p, q) d\tau \\ &= \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) (c)_{np} (d)_{nq}}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta) \Gamma(\alpha n + \beta)} \frac{\omega^n}{n!} \left( \int_a^x (\tau - a)^{\mu-1} (x - \tau)^{\beta+\alpha n-1} d\tau \right) \\ &\quad \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) (c)_{np} (d)_{nq}}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta) \Gamma(\alpha n + \beta)} \frac{\omega^n}{n!} I_{a+}^{\alpha n + \beta} [(\tau - a)^{\mu-1}] \\ &= (x - a)^{\beta+\mu-1} \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) (c)_{np} (d)_{nq}}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta) \Gamma(\alpha n + \beta)} \frac{[\omega(x - a)^\alpha]^n \Gamma(\mu) \Gamma(\alpha n + \beta)}{n! \Gamma(\alpha n + \beta + \mu)} \\ &= (x - a)^{\beta+\mu-1} \Gamma(\mu) E_{\alpha,\beta+\mu}^{\gamma,\delta;c,d}(\omega(x - a)^\alpha; p, q). \end{aligned}$$

This completes the desired proof. \_

**Theorem 2.3.2** If  $\gamma, \alpha, \beta, \omega, c \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(c) > 0$ , then  $\| \epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} \varphi \|_1 \leq B \|\varphi\|_1$ , where

$$B = (b - a)^{R(\beta)} \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) |(c)_{np}| |(d)_{nq}| |\omega(b - a)^\alpha|^n}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta) \Gamma(\alpha n + \beta) (R(\beta) + R(\alpha)n) n!} \tag{1.1.25}$$

**Proof** From (1.1.19) and (1.1.10) and by changing the order of integration by using the Dirichlet formula [18], we have

$$\begin{aligned} \| \epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} \varphi \|_1 &= \int_a^b \left| \int_a^x (x - \tau)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(x - \tau)^\alpha; p, q) \varphi(\tau) d\tau \right| dx \\ &\leq \int_a^b \left[ \int_a^x (x - \tau)^{R(\beta)-1} \left| E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(x - \tau)^\alpha; p, q) \right| dx \right] |\varphi(\tau)| d\tau. \end{aligned}$$

Substituting  $(x - \tau) = u$ , we obtain

$$= \int_a^b \left[ \int_0^{b-\tau} (u)^{R(\beta)-1} \left| E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(u)^\alpha; p, q) \right| du \right] |\varphi(\tau)| d\tau.$$

After simplification, we have

$$\| \epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} \varphi \|_1 \leq \int_a^b \left[ \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) |(c)_{np}| |(d)_{nq}| |\omega^n|}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta) \Gamma(\alpha n + \beta) n!} \left( \frac{(u)^{R(\beta)+R(\alpha)n}}{R(\beta)+R(\alpha)n} \right)_0^{b-a} \right] |\varphi(\tau)| d\tau.$$

This can also be written as

$$\begin{aligned} \| \epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} \varphi \|_1 &\leq \\ &(b - a)^{R(\beta)} \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) |(c)_{np}| |(d)_{nq}| |\omega(b - a)^\alpha|^n}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta) \Gamma(\alpha n + \beta) (R(\beta) + R(\alpha)n) n!} \cdot \int_a^b |\varphi(\tau)| d\tau \\ &= B \|\varphi\|_1, \end{aligned}$$

where

$$B = (b - a)^{R(\beta)} \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) B_q(\delta + n, d - \delta) |(c)_{np}| |(d)_{nq}| |\omega(b - a)^\alpha|^n}{B_p(\gamma, c - \gamma) B_q(\delta, d - \delta) \Gamma(\alpha n + \beta) (R(\beta) + R(\alpha)n) n!}. \tag{1.1.26}$$

This completes the desired proof.

**Theorem 2.3.3** If  $\lambda, \gamma, \alpha, \beta, \omega \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\gamma) > 0, \Re(\lambda) > 0$  and  $x > a$ , then

$$(I_{a+}^\lambda [\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} f])(x) = (\epsilon_{a+;\alpha,\beta+\mu}^{\omega;\gamma,\delta;c,d} f)(x) = (\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} [I_{a+}^\lambda f])(x) \tag{1.1.27}$$

holds for any function  $f \in L(\alpha, \beta)$ .

**Proof** From (1.1.19) and (1.1.11), we have

$$\begin{aligned} (I_{a+}^{\lambda} [\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} f])(x) &= \frac{1}{\Gamma(\lambda)} \int_a^x \frac{\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} f(\tau)}{(x-\tau)^{1-\mu}} d\tau \\ &= \frac{1}{\Gamma(\lambda)} \int_a^x (x-\tau)^{\lambda-1} \times \left\{ \int_a^{\tau} (\tau-u)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(\tau-u)^{\alpha}; p, q) f(u) du \right\} d\tau. \end{aligned}$$

By interchanging the order of integration and applying the Dirichlet formula [19], we have

$$(I_{a+}^{\lambda} [\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} f])(x) = \int_a^x \left[ \frac{1}{\Gamma(\lambda)} \int_u^x (x-\tau)^{\lambda-1} (\tau-u)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(\tau-u)^{\alpha}; p, q) d\tau \right] f(u) du.$$

Substituting  $(\tau-u) = \rho$ , we obtain

$$\begin{aligned} (I_{a+}^{\lambda} [k\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} f])(x) &= \int_a^x \left[ \frac{1}{\Gamma(\lambda)} \int_0^{x-u} (x-u-\rho)^{\lambda-1} (\rho)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(\rho)^{\alpha}; p, q) d\rho \right] f(u) du \\ &= \int_a^x \left[ \frac{1}{\Gamma(\lambda)} \int_0^{x-u} (\rho)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(\rho)^{\alpha}; p, q) (x-u-\rho)^{1-\lambda} d\rho \right] f(u) du. \end{aligned} \tag{1.1.28}$$

By the use of (1.1.11) and applying (1.1.21), we get

$$\begin{aligned} (I_{a+}^{\lambda} [\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} f])(x) &= \int_a^x [(\rho)^{\lambda+\beta-1} E_{\alpha,\beta+\lambda}^{\gamma,\delta;c,d}(\omega(\rho)^{\alpha}; p, q)] f(u) du \\ &= \int_a^x (x-u)^{\lambda+\beta-1} E_{\alpha,\beta+\lambda}^{\gamma,\delta;c,d}(\omega(x-u)^{\alpha}; p, q) f(u) du ; \end{aligned}$$

thus, we get

$$(I_{a+}^{\lambda} [\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} f])(x) = (\epsilon_{a+;\alpha,\beta+\lambda}^{\omega;\gamma,\delta;c,d} f)(x), \tag{1.1.29}$$

this is the required proof of (1.1.27).

To prove the second part, we begin from the right-hand side of (1.1.27) and using (1.1.19), we have

$$\begin{aligned} (\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} [I_{a+}^{\lambda} f])(x) &= \int_a^x (x-\tau)^{\beta-1} E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(x-\tau)^{\alpha}; p, q) [I_{a+}^{\lambda} f](\tau) d\tau \\ &= \int_a^x E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(x-\tau)^{\alpha}; p, q) \left( \frac{1}{\Gamma(\lambda)} \int_a^{\tau} \frac{f(u)}{(x-u)^{1-\mu}} du \right) d\tau. \end{aligned}$$

By interchanging the order of integration and using the Dirichlet formula [18], we obtain

$$(\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} [I_{a+}^{\lambda} f])(x) = \int_a^x \frac{1}{\Gamma(\lambda)} \left[ \int_u^x (x-\tau)^{\beta-1} (\tau-u)^{\lambda-1} E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(x-\tau)^{\alpha}; p, q) d\tau \right] \times f(u) du.$$

Substituting  $(x-\tau) = \rho$

$$\begin{aligned} (\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} [I_{a+}^{\lambda} f])(x) &= \int_a^x \frac{1}{\Gamma(\lambda)} \left[ \int_{x-u}^0 (\rho)^{\beta-1} (x-\rho-u)^{\lambda-1} E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(\rho)^{\alpha}; p, q) (-d\rho) \right] \times f(u) du \\ &= \int_a^x \frac{1}{\Gamma(\lambda)} \left[ \int_0^{x-u} (\rho)^{\beta-1} (x-\rho-u)^{\lambda-1} E_{\alpha,\beta}^{\gamma,\delta;c,d}(\omega(\rho)^{\alpha}; p, q) d\rho \right] \times f(u) du. \end{aligned}$$

Again by making use of (1.1.11) and applying (1.1.21), we get

$$(\epsilon_{a+;\alpha,\beta}^{\omega;\gamma,\delta;c,d} [I_{a+}^{\lambda} f])(x) = (\epsilon_{a+;\alpha,\beta+\lambda}^{\omega;\gamma,\delta;c,d} f)(x).$$

Thus (1.1.29) and (1.1.30) complete the desired proof of (1.1.27).

### III. SPECIAL CASES

If we put  $\delta = q = 1$  in Theorem 2.2.2 then we get following result.

**Theorem 2.4.1** Suppose  $x > a (\alpha \in \mathcal{R}_+ = [0, \infty)), \gamma, \lambda, \beta, \omega \in \mathbb{C}, \Re(\beta) > 0, \Re(\lambda) > 0$ ; then

$$I_{a+}^{\lambda} \left[ (\tau-a)^{\beta-1} E_{\alpha,\beta}^{\gamma,c}(\omega(\tau-a)^{\alpha}; p) \right] (x) = (x-a)^{\lambda+\beta-1} E_{\alpha,\beta+\lambda}^{\gamma,c}(\omega(x-a)^{\alpha}; p)$$

$$D_{a+}^{\lambda} \left[ (\tau-a)^{\beta-1} E_{\alpha,\beta}^{\gamma,c}(\omega(\tau-a)^{\alpha}; p) \right] = (x-a)^{\beta-\lambda-1} E_{\alpha,\beta-\lambda}^{\gamma,c}(\omega(x-a)^{\alpha}; p)$$

and

$$D_{a+}^{\lambda,\nu} \left[ (\tau-a)^{\beta-1} E_{\alpha,\beta}^{\gamma,c}(\omega(\tau-a)^{\alpha}; p) \right] (x) = (x-a)^{\beta-\lambda-1} E_{\alpha,\beta-\lambda}^{\gamma,c}(\omega(x-a)^{\alpha}; p).$$

If we put  $\gamma = p = \delta = q = 1$  in theorem 2.2.2 then we get following result.

**Theorem 2.4.2** Suppose  $x > a (\alpha \in \mathcal{R}_+ = [0, \infty)), \gamma, \lambda, \beta, \omega \in \mathbb{C}, \Re(\beta) > 0, \Re(\lambda) > 0$ ; then

$$I_{a+}^{\lambda} \left[ (\tau-a)^{\beta-1} E_{\alpha,\beta}(\omega(\tau-a)^{\alpha}) \right] (x) = (x-a)^{\lambda+\beta-1} E_{\alpha,\beta+\lambda}(\omega(x-a)^{\alpha})$$

$$D_{a+}^{\lambda} \left[ (\tau-a)^{\beta-1} E_{\alpha,\beta}(\omega(\tau-a)^{\alpha}) \right] = (x-a)^{\beta-\lambda-1} E_{\alpha,\beta-\lambda}(\omega(x-a)^{\alpha})$$

and



$$D_{a+}^{\lambda, \nu} [(\tau - a)^{\beta-1} E_{\alpha, \beta}(\omega(\tau - a)^\alpha)](x) = (x - a)^{\beta-\lambda-1} E_{\alpha, \beta-\lambda}(\omega(x - a)^\alpha).$$

If we take  $\delta = d = 1$  in Definition 2.1.1 then we get the following definition

**Definition 2.4.3** If  $\gamma, \omega \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0$ , then

$$\left( \epsilon_{a+; \alpha, \beta}^{\omega; \gamma; c} f \right) (x) = \int_a^x (x - \tau)^{\beta-1} E_{\alpha, \beta}^{\gamma; c}(\omega(x - \tau)^\alpha; p) f(\tau) d\tau,$$

where  $x > a$ . Substituting  $p = 0$ , then (19) reduces to the operator

$$\left( \epsilon_{a+; \alpha, \beta}^{\omega; \gamma} f \right) (x) = \int_a^x (x - \tau)^{\beta-1} E_{\alpha, \beta}^{\gamma, \delta}(\omega(x - \tau)^\alpha) f(\tau) d\tau,$$

If we take  $\gamma = c = \delta = d = 1$  in Definition 2.1.1 then we get the following definition

**Definition 2.4.4** If  $\gamma, \omega \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0$ , then

$$\left( \epsilon_{a+; \alpha, \beta}^{\omega} f \right) (x) = \int_a^x (x - \tau)^{\beta-1} E_{\alpha, \beta}(\omega(x - \tau)^\alpha) f(\tau) d\tau$$

If we take  $\delta = d = 1$  in Definition 2.3.1 then we get the following results

**Theorem 2.4.5** If  $\gamma, \omega \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\lambda) > 0$  and  $\Re(\beta) > 0$ , then

$$\left( \epsilon_{a+; \alpha, \beta}^{\omega; \gamma; c} [(\tau - a)^{\mu-1}] \right) (x) = (x - a)^{(\mu+\beta-1)} \Gamma(\mu) E_{\alpha, \beta+\mu}^{\gamma; c}(\omega(x - a)^\mu; p) f(\tau) d\tau.$$

If we take  $\gamma = c = \delta = d = 1$  in Theorem 2.3.2 then we get the following results

**Theorem 2.4.6** If  $\gamma, \omega \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\lambda) > 0$  and  $\Re(\beta) > 0$ , then

$$\left( \epsilon_{a+; \alpha, \beta}^{\omega} [(\tau - a)^{\mu-1}] \right) (x) = (x - a)^{(\mu+\beta-1)} \Gamma(\mu) E_{\alpha, \beta+\mu}(\omega(x - a)^\mu) f(\tau) d\tau.$$

If we take  $\delta = d = 1$  in Theorem 2.3.2 then we get the following results

**Theorem 2.4.7** If  $\gamma, \alpha, \beta, \omega, c \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(c) > 0$ , then

$$\left\| \epsilon_{a+; \alpha, \beta}^{\omega; \gamma; c} \varphi \right\|_1 \leq B \|\varphi\|_1,$$

where

$$B = (b - a)^{R(\beta)} \sum_{n=0}^{\infty} \frac{B_p(\gamma + n, c - \gamma) |c|_{np}}{B_p(\gamma, c - \gamma) \Gamma(\alpha n + \beta) (R(\beta) + R(\alpha)n)} \frac{|\omega(b - a)^\alpha|^n}{n!}.$$

If we take  $\gamma = c = \delta = d = 1$  in Theorem 2.3.2 then we get the following results

**Theorem 2.4.8** If  $\lambda, \gamma, \alpha, \beta, \omega \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\gamma) > 0, \Re(\lambda) > 0$  and  $x > a$ , then

$$\left( I_{a+}^\lambda \left[ \epsilon_{a+; \alpha, \beta}^{\omega} f \right] \right) (x) = \left( \epsilon_{a+; \alpha, \beta+\mu}^{\omega} f \right) (x) = \left( \epsilon_{a+; \alpha, \beta}^{\omega} \left[ I_{a+}^\lambda f \right] \right) (x)$$

holds for any function  $f \in L(\alpha, \beta)$ .

## REFERENCES

1. Agarwal, P., Choi, J., Jain, S., Rashidi, M.M.: Certain integrals associated with generalized Mittag–Leffler function. Commun. Korean Math. Soc. 32(1), 29–38 (2017)
2. Agarwal, P., Nieto, J.J.: Some fractional integral formulas for the Mittag–Leffler type function with four parameters. Open Math. 13(1), 537–546 (2015)
3. Agarwal, P., Chand, M., Jain, S.: Certain integrals involving generalized Mittag–Leffler functions. Proc. Nat. Acad. Sci. India Sect. A 85(3), 359–371 (2015)
4. Agarwal, P., Rogosin, S.V., Trujillo, J.J.: Certain fractional integral operators and the generalized multi-index Mittag–Leffler functions. Proc. Indian Acad. Sci. Math. Sci. 125(3), 291–306 (2015)
5. Camargo, R.F., Capelas de Oliveira, E., Vas, J.: On the generalized Mittag–Leffler function and its application in a fractional telegraph equation. Math. Phys. Anal. Geom. 15(1), 1–16 (2012)
6. Chaudhry, M.A., Qadir, A., Srivastava, H.M., Paris, R.B.: Extended hypergeometric and confluent hypergeometric functions. Appl. Math. Comput. 159, 589–602 (2004)
7. Džrbašjan, M.M.: Integral transforms and representations of functions in the complex domain. Nauka, Moscow (1966) (in Russian)
8. Gorenflo, R., Mainardi, F.: Fractional calculus: integral and differential equations of fractional order. In: Carpinteri A., Mainardi, F. (eds.) Fractals and Fractional Calculus in Continuum Mechanics. Springer Series on CSM Courses and Lectures, vol. 378, pp. 223–276 (1997)



9. Gorenflo, R., Mainardi, F., Srivastava, H.M.: Special functions in fractional relaxation-oscillation and fractional diffusion-wave phenomena. In: Bainov, D. (ed.) Proceedings of the Eighth International Colloquium on Differential Equations (Plovdiv, Bulgaria; August 18–23, 1997), pp. 195–202. VSP Publishers, Utrecht and Tokyo (1998)
10. Gorenflo, R., Kilbas, A.A., Rogosin, S.V.: On the generalized Mittag–Leffler type functions. *Integral Transform. Spec. Funct.* 7, 215–224 (1998)
11. Gorenflo, R., Luchko, Y., Mainardi, F.: Wright functions as scale-invariant solutions of the diffusionwave equation. *J. Comput. Appl. Math.* 118, 175–191 (2000)
12. Hilfer, R. (ed.): Applications of Fractional Calculus in Physics. World Scientific Publishing Company, Singapore (2000)
13. Hilfer, R.: Fractional time evolution. In: Hilfer, R. (ed.) Applications of Fractional Calculus in Physics. World Scientific Publishing Company, Singapore (2000)
14. Hilfer, R., Seybold, H.: Computation of the generalized Mittag–Leffler function and its inverse in the complex plane. *Integral Transform. Spec. Funct.* 17, 637–652 (2006)
15. Kilbas, A.A., Saigo, M.: On Mittag–Leffler type function, fractional calculus operators and solutions of integral equations. *Integral Transform. Spec. Funct.* 4, 355–370 (1996)
16. Kilbas, A.A., Srivastava, H.M., Trujillo, J.J.: Theory and Applications of Fractional Differential Equations, North-Holland Mathematical Studies, vol. 204. Elsevier (North-Holland) Science Publishers, Amsterdam (2006)
17. Mathai, A.M., Haubold, H.J.: Special Functions for Applied Scientists. Springer, Berlin (2010)
18. Miller, K.S., Ross, B.: An Introduction to the Fractional Calculus and Fractional Differential Equations. Wiley, New York (1993)
19. Özarıslan, M.A., Yılmaz, B.: The extended Mittag–Leffler function and its properties. *J Inequal Appl* 2014, 85 (2014)
20. Prabhakar, T.R.: A singular integral equation with a generalized Mittag–Leffler function in the kernel. *Yokohama Math. J.* 19, 7–15 (1971)
21. Rao, S.B., Prajapati, J.C., Patel, A.K., Shukla, A.K.: Some properties of wright-type hypergeometric function via fractional calculus. *Adv. Differ. Equat.* 2014, Art. no. 119 (2014)
22. Saigo, M., Kilbas, A.A.: On Mittag–Leffler type function and applications. *Integral Transform. Spec. Funct.* 7, 97–112 (1998)
23. Samko, S.G., Kilbas, A.A., Marichev, O.I.: Fractional Integrals and Derivatives: Theory and Applications. Gordon and Breach Science Publishers, Yverdon (1993)
24. Seybold, H.J., Hilfer, R.: Numerical results for the generalized Mittag–Leffler function. *Fract. Calc. Appl. Anal.* 8, 127–139 (2005)
25. Srivastava, H.M., Tomovski, Ž.: Fractional calculus with an integral operator containing a generalized Mittag–Leffler function in the kernel. *Appl. Math. Comput.* 211, 198–210 (2009)
26. Srivastava, H.M., Manocha, H.L.: A Treatise on Generating Functions. Wiley/Ellis Horwood, New York/Chichester (1984)



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