



e-ISSN: 2319-8753 | p-ISSN: 2347-6710

IJRSET

International Journal of Innovative Research in
SCIENCE | ENGINEERING | TECHNOLOGY

A futuristic digital interface with a glowing Earth globe in the center. The background is dark blue with various data visualizations including bar charts, line graphs, and network diagrams. Text elements like 'BUSINESS', 'NETWORK SEARCH', and 'WORLD' are visible. A person's hand is seen at the bottom, interacting with the interface.

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

Volume 12, Issue 7, July 2023

ISSN INTERNATIONAL
STANDARD
SERIAL
NUMBER
INDIA

Impact Factor: 8.423

 9940 572 462

 6381 907 438

 ijrset@gmail.com

 www.ijrset.com

ON THE TERNARY CUBIC DIOPHANTINE EQUATION

$$9(x^2 + y^2) - 16xy = 528z^3$$

V.Krithika¹, A.Vijayasankar² and M.A.Gopalan³

¹ Assistant Professor, Department of Mathematics, National College, Trichy, Tamilnadu, India

² Associate Professor, Department of Mathematics, National College, Trichy, Tamilnadu, India

³ Professor, Dept. of Mathematics, SIGC, Trichy, Tamilnadu, India

ABSTRACT:

The sequences of integral solutions to the cubic equation with three variables $9(x^2 + y^2) - 16xy = 528z^3$ are obtained.

KEYWORDS:

Non-Homogeneous Cubic, Cubic with three unknowns, integral solutions.

Introduction:

The Diophantine equation offer an unlimited field for research due to their variety [1-3]. In particular, one may refer [1-6] for cubic equation with three unknowns are studied for its non-trivial integral solutions. This communication concerns with the problem of obtaining infinitely many non-zero distinct integral solutions of ternary cubic equation given by $9(x^2 + y^2) - 16xy = 528z^3$.

METHOD OF ANALYSIS

The non-homogeneous ternary cubic diophantine equation to be solved for its non-zero distinct integral solutions is given by

$$9(x^2 + y^2) - 16xy = 528z^3 \tag{1}$$

Introducing the transformations

$$x = 2(u + v), y = 2(u - v), u \neq v \neq 0 \tag{2}$$

in (1), it leads to

$$u^2 + 17v^2 = 66z^3 \tag{3}$$

Which is to be solved through different methods to obtain different sets of integer solutions to (1).

Method 1:

$$\text{Assume } z = a^2 + 17b^2 \quad (4)$$

where a and b are non-zero integers.

$$\text{Write 66 as } 66 = (7 + i\sqrt{17})(7 - i\sqrt{17}) \quad (5)$$

Substituting (4) & (5) in (3) and applying the method of factorization, define

$$u + i\sqrt{17}v = (7 + i\sqrt{17})(a + i\sqrt{17}b)^3$$

Equating the real and imaginary parts, we have

$$\begin{aligned} u &= 7a^3 + 289b^3 - 51a^2b - 357ab^2 \\ v &= a^3 - 119b^3 + 21a^2b - 51ab^2 \end{aligned} \quad (6)$$

From (2), we have

$$\begin{aligned} x(a, b) &= 2\{8a^3 + 170b^3 - 408ab^2 - 30a^2b\} \\ y(a, b) &= 2\{6a^3 + 408b^3 - 306ab^2 - 72a^2b\} \end{aligned} \quad (7)$$

Note 1:

$$\text{Rewrite 66 as } 66 = \frac{(73 + i\sqrt{17})(73 - i\sqrt{17})}{81} \quad (8)$$

Substituting (4) & (8) in (3) and applying the same procedure as mentioned in the above set, we have

$$u + i\sqrt{17}v = \frac{(73 + i\sqrt{17})}{9}(a + i\sqrt{17}b)^3$$

Equating the real and imaginary parts, we have

$$\begin{aligned} u &= \frac{1}{9}(73a^3 + 289b^3 - 51a^2b - 3723ab^2) \\ v &= \frac{1}{9}(a^3 - 1241b^3 + 219a^2b - 51ab^2) \end{aligned}$$

As our aim is to find integer solutions, choose $a = 9A, b = 9B$ in the above equations. In view of (4) & (2), the corresponding non-zero integer solutions of (1) are given by

$$\begin{aligned}x(A, B) &= 2\{5994A^3 - 77112B^3 + 13608A^2B - 305694AB^2\} \\y(A, B) &= 2\{5832A^3 + 123930B^3 - 21870A^2B - 297432AB^2\} \\z(A, B) &= 81(A^2 + 17B^2)\end{aligned}$$

Note 2:

We can write 66 in an another way as $66 = \frac{(53 + i5\sqrt{17})(53 - i5\sqrt{17})}{49}$

As proceeding in the above sets, we get the corresponding non-zero distinct integer solutions of (1) to be

$$\begin{aligned}x(A, B) &= 2\{2842A^3 + 26656B^3 - 4704A^2B - 144942AB^2\} \\y(A, B) &= 2\{2352A^3 + 114954B^3 - 20286A^2B - 119952AB^2\} \\z(A, B) &= 49(A^2 + 17B^2)\end{aligned}$$

Method 2:

One may write (3) as

$$u^2 + 17v^2 = 66z^3 * 1 \tag{9}$$

Write 1 as

$$1 = \frac{(8 + i\sqrt{17})(8 - i\sqrt{17})}{81} \tag{10}$$

Using (4), (5) and (10) in (9) and applying the method of factorization, define

$$(u + i\sqrt{17}v) = \frac{1}{9} \left[(7 + i\sqrt{17})(8 + i\sqrt{17})(a + i\sqrt{17}b)^3 \right] \tag{11}$$

Equating real and imaginary parts of (9), we have

$$\begin{aligned}u &= \frac{1}{9}(39a^3 + 4335b^3 - 765a^2b - 1989ab^2) \\v &= \frac{1}{9}(15a^3 - 663b^3 + 117a^2b - 765ab^2)\end{aligned}$$

As our aim is to find integer solutions, choosing $a = 3A, b = 3B$ in the above equations we obtain.

In view of (4) & (2), the corresponding non-zero integer solutions of (1) are given by

$$\begin{aligned}x(A, B) &= 2\{162A^3 + 11016B^3 - 1944A^2B - 8262AB^2\} \\y(A, B) &= 2\{72A^3 + 14994B^3 - 2646A^2B - 3672AB^2\} \\z(A, B) &= 9(A^2 + 17B^2)\end{aligned}$$

Note 3:

We can write 1 in an another way as $1 = \frac{(4 + i3\sqrt{17})(4 - i3\sqrt{17})}{169}$

As proceeding in the above sets, we get the corresponding non-zero distinct integer solutions of (1) given by

$$\begin{aligned} x(A, B) &= 2 \{ 338A^3 + 1287104B^3 - 227136A^2B - 17238AB^2 \} \\ y(A, B) &= 2 \{ -8112A^3 + 1154946B^3 - 203814A^2B + 413712AB^2 \} \\ z(A, B) &= 169(A^2 + 17B^2) \end{aligned}$$

Note 4:

We can write 1 in an another way as $1 = \frac{(4 + i5\sqrt{17})(4 - i5\sqrt{17})}{441}$

As proceeding in the above sets, we get the corresponding non-zero distinct integer solutions of (1) given by

$$\begin{aligned} x(A, B) &= 2 \{ -7938A^3 + 5397840B^3 - 952560A^2B + 404838AB^2 \} \\ y(A, B) &= 2 \{ -42336A^3 + 4543182B^3 - 801738A^2B + 2159136AB^2 \} \\ z(A, B) &= 441(A^2 + 17B^2) \end{aligned}$$

Method 3:

In addition to (2), let us consider another set of transformations as

$$x = 2(u + v), y = 2(u - v), z = v, u \neq v \neq 0 \tag{13}$$

$$\text{Thus, from (1) we get } u^2 = v^2(66v - 17) \tag{14}$$

After performing a few calculations, we note that (14) is satisfied by

$$\begin{aligned} v &= 1 + \frac{1}{6}(k^2 - k) \{ 11(k^2 - k) + 14 \} \\ u &= (11k^2 - 11k + 7) \left[1 + \frac{1}{6}(k^2 - k) \{ 11(k^2 - k) + 14 \} \right] \end{aligned}$$

By substituting the above u and v values in (13), we get the corresponding non-zero integer solutions to (1).

Conclusion:

In this paper, we have presented sets of infinitely many non-zero distinct integer solutions to the ternary cubic equation given by $9(x^2 + y^2) - 16xy = 528z^3$. As Diophantine equations are rich



in variety due to their definition, one may attempt to find integer solutions to higher degree diophantine equations with multiple variables along with suitable properties..

REFERENCES

1. L.J.Mordell, Diophantine Equations, Academic press, New York, 1969.
2. Carmichael, R.D. 1959. The Theory of Numbers and Diophantine Analysis, New York, Dover.
3. Dickson, L.E. 2005. History of Theory of Numbers, vol.2, Diophantine Analysis, New York, Dover.
4. S.Vidhyalakshmi, J.Shanthi, K.Hema, M.A.Gopalan, Observation on the paper entitled Integral Solution of the homogeneous ternary cubic equation $x^3 + y^3 = 52(x+y)z^3$, EPRA IJMR, Vol.8, Issue 2, 266-273, 2022.
5. M.A.Gopalan, S.Vidhyalakshmi, J.Shanthi, J.Maheswari, On ternary Cubic Diophantine equation $3(x^2 + y^2) - 5xy + x + y + 1 = 12z^3$, IJAR, Vol I, Issue 8, 209-212, 2015.
6. Manju Somanath, J.Kannan, K.Raja, Families of solution of a cubic Diophantine equation, IJRASET, Vol 4, Issue XI, Nov 2016.
7. Manju Somanath, T.Anupreethi, V.Sangeetha, Lattice points for the quadratic Diophantine equation $21(x^2 + y^2) - 19xy = 84z^2$, IJIRSET, Vol 11, Issue 5, 5542-5549, May 2022.



INNO SPACE
SJIF Scientific Journal Impact Factor
Impact Factor: 8.423



ISSN INTERNATIONAL
STANDARD
SERIAL
NUMBER
INDIA



INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

9940 572 462 6381 907 438 ijirset@gmail.com



www.ijirset.com

Scan to save the contact details