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Electrorheological Fluids: Mathematical Theory and Abstract Algebra

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ABSTRACT: In recent years, Electrorheological Fluids have been paid more attention because of their smart nature. This article presents the mathematical behaviour of electrorheological fluid. Electrorheological fluid is a special viscous fluid, consisting of solid electric particles and that shows significant changes. This article provides mathematical abstract algebra for electric particles. This article also explains in detail the mathematical background of ER fluids, which are derived from material properties, physics of ER fluid and potential real life applications

KEYWORDS: Electrorheological Fluids, Electrically charged particles, Abstract Algebra.

I. INTRODUCTION

A smart fluid is a material whose properties can be altered by applying an external electromagnetic force. An electrorheological fluid belongs to a class of smart materials and when subjected to a electric force, these fluids greatly increase its apparent viscosity. Electrorheological (ER) fluid is a special viscous fluid, consisting of solid electric particles dispersed in an insulating base fluid, and that are experience significant changes in their rheological and mechanical properties, when an electric force is applied [1,2]. The property of the ER fluids can be exploited in various technological applications. They have broad potential applications in dampers, actuators, clutches, valves, etc [3]. Generally, the rheological properties of an ER fluid along with viscosity, shear stress and internal arrangement, either decrease or increase with the applied electric force strength, based on the size and nature of the electric particles dispersed in base fluids [4,5,6]. Such kind of response takes place in millisecond scale.

ER fluids can be modeled in different many ways. The numerical simulations and continuum mechanics taking into account the dynamics and art of the particles in base fluids. The behaviour and properties of ER fluids are greatly influenced by variable electric force and in the most of the models electric force is considered as variable and to be measured for getting complete solution of the model [1,7,8,9].

II. RHEOLOGICAL VECTOR SPACE FOR ER FLUIDS

Mathematical behaviour of Electrorheological fluids can be described in different manner based on continuum mechanics, direct numerical simulation, dynamics and interaction of particles and microstructure of base fluid. In the functional relation for electrorheological fluids is developed through generalized Lebesgue and generalized Sobolev space [4,5]. In this section we carry out the mathematical formulation for ER fluids, which builds on idea of linear algebra which is the part of abstract algebra [10,11,12].

In this section, mathematical linear algebra is proposed for electrorheological fluids. This approach tries to describe the response of an electrorheological fluid under different applied forces [5]. Here we are interested to describe nature of ER fluid through linear algebra when electric force is applied.

Electrorheological fluid is the mathematical non-linear function of electric field, viscosity, yield stress, density of particles, velocity of particles, particle size, deformation stress, temperature, pressure, etc. some of the parameters are dependent and independent. The behaviour of the ER fluid under flow and subjected to an electric field, though it indicates that the some of the parameters might be vectors in the structure formation within the suspension.



Many of the variables used in electrorheological fluid flow obey the axioms of a finite dimensional linear vector space with an Inner Product, which is known as Inner Product Space (I.P.S) [10,11,12]. A result that is stated in terms of an arbitrary IPS can be applied in many contexts, achieving both generality and rheological nature of electrorheological fluid. For the purposes of this work, scalars can be assumed to be real numbers.

Vector Space for ER Fluids

Let V be a non-void set of ER fluid with two binary operations:

- i) Vector addition: This assigns to any $u, v \in V$ a sum $u + v$ in V .
- ii) Scalar Multiplication: This assigns to any $u \in V, k \in K$ a product $ku \in V$.

Then V is known as a vector space for electrorheological fluid (over the scalar field K) if the following conditions hold for any vectors $u, v, w \in V$:

[A1] $(u + v) + w = u + (v + w)$

[A2] There is a vector in V , denoted by 0 and called the zero vector, such that, for any $u \in V$,

$$u + 0 = 0 + u = u$$

[A3] For each $u \in V$, there is a vector in V , denoted by $-u$ and called the negative of u , such that ,

$$u + (-u) = (-u) + u = 0$$

[A4] $u + v = v + u$

[M1] $k(u + v) = ku + kv$ for any $k \in K$

[M2] $(a + b)u = au + bu$ for any $a, b \in K$

[M3] $(ab)u = a(bu)$ for any $a, b \in K$

[M4] $1u = u$ for the unit scalar $1 \in K$

The above axioms naturally split into two sets. The first four are only concerned with the additive structure of V . On the other hand, the remaining four axioms are concerned with the action of the field of scalars on the vector space V .

Scalar Field for ER Fluids

Suppose K is a non-void set of ER fluid mapped with two binary operations '+' and '.' called addition and multiplication i.e. for all $a, b \in K$ we have $a + b \in K$ & $a.b \in K$.

Then the algebraic structure $(K, +, .)$ is called a scalar field for ER fluid, if the following postulates are satisfied;

[K1] $a + b = b + a$ for all $a, b \in K$

[K2] $(a + b) + c = a + (b + c)$ for all $a, b, c \in K$

[K3] there exists element 0 in K such that $a + 0 = a$ for all $a \in K$

[K4] to each element a in K there exists an element $-a$ in K such that $a + (-a) = 0$

[K5] $a.b = b.a$ for all $a, b \in K$

[K6] $(a.b).c = a.(b.c)$ for all $a, b, c \in K$

[K7] there exists element 1 a non zero element denoted by 1 in K such that $a.1 = a$ for all $a \in K$

[K8] to every non zero element a in K there exists an element a^{-1} in K such that $a + a^{-1} = 0$

[K9] $a.(b + c) = a.b + a.c$ for all $a, b, c \in K$

Linear Combination, Spanning Sets

Let V is a vector space for ER fluid over the scalar field K .

A vector u in V is a linear combination of vectors $u_1, u_2, u_3, \dots, u_n$ in V if there exists scalars $a_1, a_2, a_3, \dots, a_n$ such that

$$v = a_1u_1 + a_2u_2 + a_3u_3 + \dots + a_nu_n \dots (1)$$

Let V is a vector space for ER fluid over the scalar field K . A vector $u_1, u_2, u_3, \dots, u_n$ in V are said to span V or to form a spanning set of V if every v in V is a linear combination of the vectors $u_1, u_2, u_3, \dots, u_n$, that is , if there exists scalars $a_1, a_2, a_3, \dots, a_n$ in K such that,

$$v = a_1u_1 + a_2u_2 + a_3u_3 + \dots + a_nu_n \dots (2)$$

The set of linear span is generally denoted by S .

Hence,

$$S = \{u_1, u_2, u_3, \dots, u_n\} \dots (3)$$

The collection of all such linear combinations denoted by $L(S)$

Thus, we have

$$L(s) = \{a_1u_1 + a_2u_2 + a_3u_3 + \dots + a_nu_n : u_1, u_2, u_3, \dots, u_n \in V \text{ \& } a_1, a_2, a_3, \dots, a_n \in K\} \dots (4)$$



Subspaces for ER Fluids

Let V is a vector space for ER fluid over the scalar field K and let $W \subseteq V$. Then subset W is called a subspace of V if W itself is a vector space over the same scalar field K w.r.t the operations of vector addition and scalar multiplication on V .

The way in which one shows that any set W is a vector space is to show W satisfies the eight axioms of a vector space. If W is a subset of a vector space V , then some of the conditions automatically hold in W , since they already holds in V . Simple criteria for identifying subspaces follow.

If W is a subset of a vector space V . Then W is subspace of V if the following two conditions hold:

- i) The zero vector 0 belongs to W .
- ii) For every $u, v \in W, k \in K$: a) the sum $u + v \in W$ b) the multiple $ku \in W$

Linear Dependence and Independence-

The notions of linear dependence and linear combinations are closely related.

Let V is a vector space for ER fluid. A finite set $\{v_1, v_2, \dots, v_n\}$ of vector space V is said to be linearly dependent (L.D.) if there exists scalars $a_1, a_2, a_3, \dots, a_n \in K$ not all them 0 such that;

$$a_1 v_1 + a_2 v_2 + a_3 v_3 + \dots + a_n v_n = 0$$

Let V is a vector space for ER fluid. A finite set $\{v_1, v_2, \dots, v_n\}$ of vector space V is said to be linearly independent (L.I.) if every relation of the form,

$$a_1 v_1 + a_2 v_2 + a_3 v_3 + \dots + a_n v_n = 0, a_i \in K, 1 \leq i \leq n \dots (5)$$

$$\Rightarrow a_i = 0 \text{ for each } 1 \leq i \leq n \dots (6)$$

Any infinite set of vectors of V is said to be linearly independent (L.I.) if its every finite subset is linearly independent (L.I.), otherwise it is linearly dependent (L.D.).

Basis and Dimension of Vector Space of ER Fluids

A non empty subset S of a vector space V of ER fluid is said to be a basis of V if

- i) S contains linearly independent (L.I.) vectors.
- ii) S generates V i.e., $L(S) = V$

i.e., each vector in V is a linear combination of a finite number of vectors of S .

A vector space V is said to be finite dimensional vector space (F.D.V.S) if there exists (\exists) a finite subset S of V such that $V = L(S)$. The dimension of vector space V is denoted by $dim V$.

III. LINEAR MAPPING OF ER FLUIDS

In this section, our objective is to develop a mathematical framework for studying how rheological behaviour of ER fluids responds to the application of external and internal forces. In order to do this, we must develop mathematical idealizations (i.e. Linear algebra) to the “ER fluid”. Particularly, with regard to the “ER fluid”, we must construct mathematical model with their geometric and constitutive nature.

The term “ U ” used to be a mathematical abstraction of an “ERF that occurs in nature”. The ER fluid U is comprised of a set of n particles p . In a mentioned configuration of the ER fluid, each particle is located at some definite point y in 3-dimensional space. The set of all the points in 3-dimensional space, corresponding to the locations of all the particles, is the region “ V ” occupied by the ER fluid in that configuration. The ER fluid, composed of a particular set of solid particles that can adopt different configurations under the action of applied forces and so that the set occupy different regions of space under different conditions. Consider the distinction between the ER fluid, a configuration of the ER fluid, and the region the ER fluid occupies in that configuration; we make these detailed distinctions in what follows. Furthermore, note the distinction between a particle and the position in space that occupies in some configuration.

More formally, in mechanics ER fluid U is a collection of particles or elements that can be put into one-to-one correspondence with some region V of Euclidean point space. An element $p \in U$ is called a particle of system (or material point). Thus, given ER fluid U , there is requisite a mapping T that takes particles $p \in U$ into their geometric locations $y \in V$ in 3-dimensional Euclidean space:

$$y = T(p) \text{ where } p \in U, y \in V$$

The mapping T of ER fluid is called a configuration of the ER fluid U ; y is the location occupied by the particle p in the configuration T ; and V is the region occupied by the ER fluid in the configuration T . Hence we can represent $V = T(U)$.

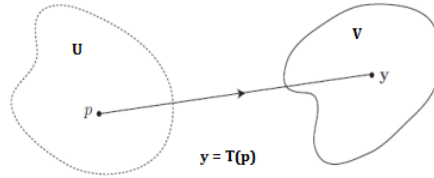


Fig.1 ER fluid U that occupies a region V in a configuration T .

Since a configuration T gives a one-to-one mapping between the particles p and positions y , there is essentially an inverse mapping T^{-1} from $V \rightarrow U$:

$$p = T^{-1}(y) \text{ where } p \in U, y \in V$$

Let U be the two vector space for the given electrorheological fluid and V be another vector space for the same electrorheological fluid over the same scalar field K . A mapping $T: U \rightarrow V$ is called a linear mapping or linear deformation of ER fluid if it satisfies the following condition;

i) $T(\alpha + \beta) = T(\alpha) + T(\beta)$ for all $\alpha, \beta \in U$ and for all $a \in K$.

ii) $T(a\alpha) = aT(\alpha)$ for all $\alpha \in U$ and for all $a \in K$.

The condition (i) and (ii) can be combined into single condition

$$T(a\alpha + b\beta) = aT(\alpha) + bT(\beta) \text{ for all } \alpha, \beta \in U \text{ and for all } a, b \in K. \dots (7)$$

Isomorphism of Electrorheological Vector Space

Let U and V be the vector space for the electrorheological fluid over the same field K . Then a mapping $T: U \rightarrow V$ is called an isomorphism of U onto V if

i) T is one one

ii) T is onto

iii) $T(a\alpha + b\beta) = aT(\alpha) + bT(\beta) \forall \alpha, \beta \in U$ and $\forall a, b \in K$.

Then the vector space V is also called the isomorphic image of the vector space U .

Kernel and Image of a Linear Mapping

Let $T: U \rightarrow V$ is a linear mapping for ER fluid. The kernel of T , written $\text{Ker}T$, is the set of elements in U that map into the zero vector 0 in V ; i.e.

$$\text{Ker}T = \{u \in U: T(u) = 0\} \dots (8)$$

The image or range of T , written $\text{Im}T$, is the set of image points in V ; i.e.

$$\text{Im}T = \{v \in V: \exists u \in U \text{ such that } T(u) = v\} \dots (9)$$

Rank and Nullity of a Linear Mapping

Let $T: U \rightarrow V$ is a linear mapping for ER fluid. Then the rank of the mapping T is defined to be the dimension of its range, and the nullity of T mapping is defined to be the dimension of its kernel.

$$\text{Rank}(T) = \dim(\text{Im}T) \ \& \ \text{Nullity}(T) = \dim(\text{Ker}T) \dots (10)$$

IV. MATRIX REPRESENTATION OF ER FLUIDS

Let U be an n dimensional vector space of electrorheological fluid over the field K and let V be an m dimensional vector space of electrorheological fluid over the field K .

Let

$$\beta = \{u_1, u_2, \dots, u_n\} \text{ and} \\ \beta' = \{v_1, v_2, \dots, v_m\}$$

be ordered bases for U and V respectively. Let T is linear mapping from U to V .

Norm or Length of a Vector in an Inner Product Space

Consider the vector space V for electrorheological fluid over the field R with standard inner product space on it. If $\alpha = (a_1, a_2, a_3) \in V(R)$, we have,

$$\langle \alpha, \alpha \rangle = a_1^2 + a_2^2 + a_3^2 \dots (15)$$

Now we know that in the three dimensional Euclidean space $\sqrt{(a_1^2 + a_2^2 + a_3^2)}$ is the length of a vector $\alpha = (a_1, a_2, a_3) \in V(R)$. Taking motivation from this fact, we make the following definition:

Definition

Let V be an I.P.S. If $\alpha \in V$, then norm or length of the vector α , written $\|\alpha\|$, is defined as the positive square root of $\langle \alpha, \alpha \rangle$ that is;

$$\|\alpha\| = \sqrt{\langle \alpha, \alpha \rangle} \dots (16)$$

In an I.P.S. a vector is called a unit vector if its length is 1.

Let V be an I.P.S. The two vectors $\alpha, \beta \in V$ are said to be orthogonal and α is said to be orthogonal to β if

$$\langle \alpha, \beta \rangle = 0 \dots (17)$$

VII. RHEOLOGICAL TENSORS

Tensors are a powerful mathematical tool that is used for derivation of constitutive equations for dynamics of fluid. The rheological tensor order is based on number of the coordinate basis. The definition of tensors is depends on multilinear algebra with a multilinear mapping. We consider the vector spaces U & V for the same electrorheological fluid. A rheological tensor can be defined as a multilinear functional R that maps vector space U into vector space V .

A set of quantities R^i , at a point $P \in U$, are called the components of contravariant rheological vector or contravariant rheological tensor of order one if they satisfy the following transformation law

$$R'^i = \frac{\partial V^i}{\partial U^j} R^j \quad \text{or} \quad R^i = \frac{\partial U^i}{\partial V^j} R'^j \dots (18)$$

The partial derivatives of co-ordinates are evaluated at the point $P \in U$

A set of quantities R_i , at a point $P \in U$, are called the components of covariant rheological vector or covariant rheological tensor of order one if they satisfy the following transformation law

$$R'_i = \frac{\partial U^j}{\partial V^i} R_j \quad \text{or} \quad R_i = \frac{\partial V^j}{\partial U^i} R'_j \dots (19)$$

A set of quantities R^{rs} , at a point $P \in U$, are called contravariant rheological tensor of order two, if they satisfy the following transformation law

$$R'^{rs} = \frac{\partial V^r}{\partial U^a} \frac{\partial V^s}{\partial U^b} R^{ab} \quad \text{or} \quad R^{rs} = \frac{\partial U^r}{\partial V^a} \frac{\partial U^s}{\partial V^b} R'^{ab} \dots (20)$$

A set of quantities R_{rs} , at a point $P \in U$, are called covariant rheological tensor of order two, if they satisfy the following transformation law

$$R'_{rs} = \frac{\partial U^a}{\partial V^r} \frac{\partial U^b}{\partial V^s} R_{ab} \quad \text{or} \quad R_{rs} = \frac{\partial V^a}{\partial U^r} \frac{\partial V^b}{\partial U^s} R'_{ab} \dots (21)$$

Similarly, the mixed tensor of order two is defined as,

$$R'^r_s = \frac{\partial V^r}{\partial U^a} \frac{\partial U^b}{\partial V^s} R^a_b \quad \text{or} \quad R^r_s = \frac{\partial U^r}{\partial V^a} \frac{\partial V^b}{\partial U^s} R'^a_b \dots (22)$$

The total number of components of a tensor can be calculated as

$$\text{Number of components of a tensor} = (\text{dimension of vector space})^{\text{order of a tensor}}$$

$$\text{Number of components of a vector (contra or co)} = n^1 = n$$

$$\text{Number of a components of a mixed tensor of order } \alpha + \beta = n^{\alpha+\beta}$$

Where n is the dimension of the vector space U .

VIII. TRANSFORMATION OF RHEOLOGICAL COORDINATES IN ER FLUIDS

In this section, we explore rheological transformation where a transformation is a function that "maps" one set to another.

Consider a system of rheological coordinates $R^i; (R^1, R^2, \dots, R^n)$ in a rheological vector space V_n of ER fluid.

Transform these variables R^i to R^j such that,

$$R^j = R^j(R^1, R^2, \dots, R^n) \quad j = 1, 2, \dots, m \quad \dots (23)$$

Here R^i is single valued continuous differentiable function of the variables R^1, R^2, \dots, R^n for their certain ranges.

These transformation equations defines a new set of co-ordinates $R^1, R^2, R^3, \dots, R^m$.

The equations (23) are solvable for R^i i.e.

$$R^i = R^i(R^1, R^2, \dots, R^m) \quad i = 1, 2, \dots, n \quad \dots (24)$$

The Jacobian of the transformation for equation (23) is defined to be,

$$J = \left| \frac{\partial R^j}{\partial R^i} \right| \neq 0$$

Differentiating equation (23), we get,

$$dR^j = \frac{\partial R^j}{\partial R^1} dR^1 + \frac{\partial R^j}{\partial R^2} dR^2 + \dots + \frac{\partial R^j}{\partial R^n} dR^n$$

$$dR^j = \sum_{i=1}^n \frac{\partial R^j}{\partial R^i} dR^i \quad , \quad j = 1, 2, \dots, m \quad \dots (25)$$

This gives the rheological transformation law of the differentiable of the coordinates i.e. the relation between dR^i and dR^j .

IX. CONCLUSION

In this article, linear algebra has been developed for ER fluids to determine constitutive relations for dependent variables like internal specific energy, potential function, yield stress, charge on particles, stress tensor, magnetic and electric polarization, and heat flux. It focuses on influence of electric field on art of electrorheological fluids. In future, we shall derive constitutive relations for ER fluids and compare them with experimental results that are available.

REFERENCES

- [1] N. M. Kuznetsov, V. V. Kovaleva, S. I. Belousov, and S. N. Chvalun. "Electrorheological fluids: from historical retrospective to recent trends." *Materials Today Chemistry*, vol. 26, 101066, 2022.
- [2] H. P. Salunkhe, S. D. Thikane, "Vector Space & Linear Mapping for Electrorheological Fluid", *Mathematical Sciences International Research Journal*, Vol. 7, pp. 61-66, 2018.
- [3] Hanmant Salunkhe, Ajit Kolekar, Surendra Thikane, "Electrorheological Fluids: Properties, Technology and Modern Applications", *International Research Journal of Engineering and Technology*, Vol. 10, Issue 2, pp. 129-134, 2023.
- [4] Tian Hao, "Electrorheological Fluids-The Non -aqueous Suspensions", Elsevier, 2005.
- [5] R. Drouot, G. Napoli, G. Racineux, "Continuum Modelling of Electrorheological Fluids", *International Journal of Modern Physics B*", Vol. 16, No. 17n18, pp. 2649-2654, 2002.
- [6] R. Tao, J. Zhang, Y. Shiroyanagi, X. Tang, X. Zhang, "Electrorheological Fluids under Shear", *International Journal of Modern Physics B*", Vol. 15, No. 06n07, pp. 918-929, 2001.
- [7] Yu. G. Yanovsky, V.E. Zgaevskii, Z.P. Shulman and E.V. Korobko, "ER-Fluid Rheological Properties in Terms of the Multiparticle Model of a Composite", *International Journal of Modern Physics B*", Vol. 16, No. 17n18, pp. 2676-2682, 2002.
- [8] Luiz B. Pompeo Neto, Antonio Celso F. Arruda, Sheila L. Vieira, "Prediction of the Dynamics of Electrorheological Fluids", *Proceeding of the 8th International conference on Electrorheological Fluids and Magnetorheological Suspensions*, World Scientific, 2002, pp 794-798.
- [9] H See, H Tamura and M Doi, "The role of water capillary forces in electro-rheological fluids", *Journal of Physics D: Applied Physics*, Volume 26(5), pp. 746-752, 1993.
- [10] David C. Lay, "Linear algebra and its Applications", Pearson Education, 2009.
- [11] Ward Cheney, David Kincaid, "Linear algebra: Theory and Applications", Jones & Bartlett Learning, 2012.
- [12] Kenneth Hoffman, Ray Alden Kunze, "Linear Algebra", Prentice-Hall of India Pvt. Limited, 1971.



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