



e-ISSN: 2319-8753 | p-ISSN: 2347-6710

IJRSET

International Journal of Innovative Research in
SCIENCE | ENGINEERING | TECHNOLOGY



INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

Volume 13, Issue 2, February 2024

ISSN INTERNATIONAL
STANDARD
SERIAL
NUMBER
INDIA

Impact Factor: 8.423

9940 572 462

6381 907 438

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Combining Electric Field Integral Equation and Lagrange Interpolation to Solve the Cavity Eigenvalue Problem

V R V T R Manikyamba¹, Vaishnavi Kadiyala², Bobbili Ekshitha Sai³

Associate Professor in Mathematics, Department of Science and Humanities, VSM College of Engineering, Ramachandrapuram, Andhra Pradesh, India¹

Student, Department of Electrical and Electronics Engineering, Shri Vishnu Engineering College for Women, Bhimavaram, Andhra Pradesh, India²

Student, Department of Electrical and Electronics Engineering, Shri Vishnu Engineering College for Women, Bhimavaram, Andhra Pradesh, India³

ABSTRACT: Among the classical issues in electromagnetics is to calculate the eigenmodes and eigenfrequencies of a perfect electric conductor cavity. Given that the frequency is the basis of the exponential function of the free space Green's function, solving the problem using the electric field integral equation (EFIE) results in a nonlinear eigenvalue problem. Usually, it is difficult to tackle a nonlinear eigenvalue issue. An iterative, straightforward approach to the EFIE eigenvalue problem is presented in this study. The polynomial eigenvalue problem can be adjusted to become the generalized eigenvalue issue by using the Lagrange interpolation of the Green's function to the nonlinear eigenvalue problem. The resonance frequencies and modes of a cavity can be found by repeatedly solving the generalized eigenvalue problem. The method can be used to cavities of any shape, and the numerical computation is simple to perform. Several numerical results are shown to illustrate the method's accuracy and viability.

KEYWORDS: EFIE, Generalized Eigenvalue, Nonlinear Eigenvalue, Lagrange Interpolation, Resonance frequencies.

I. INTRODUCTION

In the field of electromagnetics, eigenvalue difficulties are prevalent and significant. Typically, the finite element and finite difference methods are used to tackle issues of this nature. The nonlinear eigenvalue problem that results from solving the eigenvalue problem with the electric field integral equation (EFIE) is challenging to manage. Certain somewhat complex root search techniques, such as singular value decomposition, must be applied.

This paper proposes a straightforward iterative solution to the EFIE eigenvalue problem. The nonlinear eigenvalue problem is converted to a polynomial eigenvalue problem by using Lagrange interpolation to increase the Green's function. The matured eigenvalue solvers are capable of rearranging a polynomial eigenvalue problem to a generalized eigenvalue problem. The process can be repeated to find the cavity's precise resonance frequencies and modes.

II. EFIE FOR THREE-DIMENSIONAL CONDUCTING SURFACES

The EFIE for a perfectly conducting surface is

$$-\frac{j}{\omega\mu}\hat{n}(\mathbf{r})\times\mathbf{E}^i(\mathbf{r})=\hat{n}(\mathbf{r})\times\iint_S G(\mathbf{r},\mathbf{r}')\left[\mathbf{J}(\mathbf{r}')+\frac{1}{k^2}\nabla'\nabla'\cdot\mathbf{J}(\mathbf{r}')\right]d\mathbf{r}'\quad (1)$$

We discretize it and we can obtain the impedance matrix Z^- with elements given by

$$z_{mn} = \iint_{\mathbf{f}_m} \iint_{\mathbf{f}_n} [\mathbf{f}_m(\mathbf{r}) \cdot \mathbf{f}_n(\mathbf{r}') - \frac{1}{k^2} \nabla \cdot \mathbf{f}_m(\mathbf{r}) \nabla' \cdot \mathbf{f}_n(\mathbf{r}')] \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}' d\mathbf{r}. \quad (2)$$

III. EIGEN VALUE EQUATION

An ideal electric conductor cavity's eigenmodes and eigenfrequencies can only be determined by solving the nonlinear eigenvalue problem

$$\bar{\mathbf{Z}}(k)\mathbf{I} = 0, \quad (3)$$

where \mathbf{I} denotes the vector of surface current. The determinant of Z^- must equal zero in order for the equation to have nonzero solutions, i.e.

$$\det \bar{\mathbf{Z}} = 0. \quad (4)$$

Each element of Z^- is a function of k in Eq. (2) since the eigenvalue k is the argument of the exp. function $e^{-jk|\mathbf{r}-\mathbf{r}'|}$. This results in the nonlinear eigenvalue issue in Eq. (3). Convenience dictates that $R = |\mathbf{r} - \mathbf{r}'|$. Rather than tackling the nonlinear problem directly, we can utilize Lagrange interpolation to expand e^{-jkR} , which can be represented as a unary cubic polynomial, by taking three points within the range of (k_0, k_1) .

$$e^{-jkR} \approx A + Bk + Ck^2. \quad (5)$$

We define two functions

$$X(x) = - \iint_{\mathbf{f}_m} \iint_{\mathbf{f}_n} \nabla \cdot \mathbf{f}_m(\mathbf{r}) \nabla' \cdot \mathbf{f}_n(\mathbf{r}') \frac{x}{4\pi R} d\mathbf{r}' d\mathbf{r},$$

$$Y(x) = \iint_{\mathbf{f}_m} \iint_{\mathbf{f}_n} \mathbf{f}_m(\mathbf{r}) \cdot \mathbf{f}_n(\mathbf{r}') \frac{x}{4\pi R} d\mathbf{r}' d\mathbf{r}.$$

Substituting Eq. (5) into Eq. (2), we have

$$z_{mn} \approx \frac{1}{k^2} X(A) + \frac{1}{k} X(B) + X(C) + Y(A) + kY(B) + k^2 Y(C). \quad (6)$$

As we can see in Eq. (6), z_{mn} has five parts, i.e. k^{-2} term, k^{-1} term, constant term, k term, and k^2 term, so we can represent Z^- as

$$\bar{\mathbf{Z}} = k^{-2} \bar{\mathbf{Z}}_0 + k^{-1} \bar{\mathbf{Z}}_1 + \bar{\mathbf{Z}}_2 + k \bar{\mathbf{Z}}_3 + k^2 \bar{\mathbf{Z}}_4. \quad (7)$$

Substituting Eq. (7) into Eq. (3) with some rearrangement yields

$$\bar{\mathbf{Z}}_0 \mathbf{I} + k \bar{\mathbf{Z}}_1 \mathbf{I} + k^2 \bar{\mathbf{Z}}_2 \mathbf{I} + k^3 \bar{\mathbf{Z}}_3 \mathbf{I} + k^4 \bar{\mathbf{Z}}_4 \mathbf{I} = 0. \quad (8)$$

By letting

$$\mathbf{x}_0 = \mathbf{I}, \mathbf{x}_1 = k\mathbf{I}, \mathbf{x}_2 = k^2\mathbf{I}, \mathbf{x}_3 = k^3\mathbf{I}, \quad (9)$$

the polynomial eigenvalue problem can be transformed to the generalized eigenvalue problem as

$$= k \begin{pmatrix} \bar{Z}_0 & 0 & 0 & 0 \\ 0 & \bar{E} & 0 & 0 \\ 0 & 0 & \bar{E} & 0 \\ 0 & 0 & 0 & \bar{Z}_0 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = k \begin{pmatrix} \bar{Z}_1 & \bar{Z}_2 & \bar{Z}_3 & \bar{Z}_4 \\ \bar{E} & 0 & 0 & 0 \\ 0 & \bar{E} & 0 & 0 \\ 0 & 0 & \bar{E} & 0 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad (10)$$

where E^- is the identity matrix. Eq. (10) is an generalized eigenvalue problem, which can be solved by conventional methods.

We initially estimate a range (f_0, f_1) that contains the resonance frequency in order to determine the cavity's resonance frequency. Lagrange interpolation is done inside the range. All we have to do is obtain a result where the imaginary component is almost equal to zero and the real part falls inside the range. We can run iterative calculations and set a lower range to obtain a more accurate answer.

IV. NUMERICAL RESULTS

We have performed numerical calculations for a number of typical metal resonance cavities using the suggested methodology.

A. Rectangular cavity:

The dimensions of the rectangular resonance cavity are 0.3 m for length, 0.5 m for width, and 0.2 m for height. With an inaccuracy of less than 0.5%, the converged result agrees with the theoretical value. Table I indicates that there are 796 mesh triangles in total.

Fig. (1) displays the surface current's X-axis component.

TABLE I: TE₁₀₁ mode resonance frequency

Number of triangles	Theoretical value	Calculated value	Error
796	0.5831 GHz	0.5813 GHz	0.309%

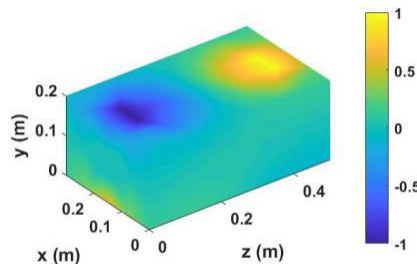


Fig. 1: X-axis component of surface current

B. Cylindrical Cavity:

The cylindrical cavity has a radius and height of 0.1 m. With an inaccuracy of 0.749%, the resonance frequency value converges to the theoretical value. Table II indicates that there are 750 mesh triangles in total. Fig. (2) displays the surface current modulus.

TABLE II: TM₀₁₀ mode resonance frequency

Number of triangles	Theoretical value	Calculated value	Error
750	1.1483 GHz	1.1569 GHz	0.749%

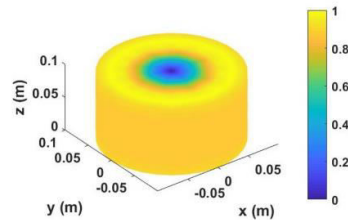


Fig. 2: Modulus of surface current

V. CONCLUSION

This work presents a straightforward, iterative solution to the EFIE eigenvalue problem. The nonlinear eigenvalue issue is first converted via Lagrange interpolation of the free space Green's function into a generalized linear eigenvalue problem. After then, it is possible to compute the resonance frequencies and modes repeatedly. The variational approach [5] can be used to determine the resonance frequency's initial guess.

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