

DOUBLE-DIFFUSIVE MARANGONI CONVECTION IN SUPERPOSED FLUID AND POROUS LAYERS

Gangadharaiah Y. H

Department of Mathematics, New Horizon College of Engineering, Bangalore 560 103, India

Abstract: The Double-Diffusive Marangoni convection in a composite system comprised of an incompressible fluid-saturated porous layer underlying a layer of same fluid is considered. The lower surface of the porous layer is rigid and the upper free surface are considered to be insulating to temperature and solutes concentration perturbations. At the upper free surface, the surface tension effects depending on temperature and both the solute concentrations are considered. At the interface, the normal and tangential components of velocity, heat and solute concentrations and their fluxes are assumed to be continuous. The resulting eigen value problem is solved using Regular perturbation technique with wave number a as perturbation parameter and the expression for the Thermal Marangoni Number is obtained. The effect of variation of different physical parameters on the same is investigated in detail.

Key words: Double diffusive convection; composite layer; solute concentration.

I. INTRODUCTION

Double diffusive convection in composite layers (two layer flow) is encountered in a wide range of industrial and geophysical applications, such as flows in fuel cells, filtration processes, the extraction of oil from underground reservoirs, ground-water pollution, and in flow in biological materials, the manufacturing of composite materials used in the aircraft and automobile industries, flow of water under the earth's surface and growing of compound films in thermal chemical vapour deposition reactors (Nield and Bejan (2006)).

Chen and Chen (1998) have considered the problem of onset of finger convection using BJ-slip condition at the interface. The problem of double diffusive convection for a thermohaline system consisting of a horizontal fluid layer above a saturated porous bed has been investigated experimentally by Poulikakos and Kazmierczak (1989). Venkatachalappa *et al* (1997) have investigated the double diffusive convection in composite layer conducive for hydrothermal growth of crystals with the lower boundary rigid and the upper boundary free with deformation.

Double diffusive convection in composite layers is investigated by Many of the researchers started by Nield (1977), Taslim and Narusawa (1989), McKay (1998), Chen (1990). Recently Shivakumara *et. al* (2006) have investigated the onset of surface tension driven convection in a two layer system comprising an incompressible fluid saturated porous layer over which lies a layer of the same fluid. The critical Marangoni number is obtained for insulating boundaries both by Regular Perturbation technique and also by exact method. They also have compared the results obtained by both the methods and found in agreement.

The aim of this paper is, therefore, to study Double-Diffusive Marangoni convection in in a composite system solely due to temperature and solutes concentration surface tension effects at the upper e free surface of a fluid layer overlying porous layer. The resulting eigen value problem is solved using Regular perturbation technique with wave number a as perturbation parameter. At the interface of the porous and fluid media, Beavers condition is used, analytic expression for the Thermal Marangoni Number is obtained Marangoni number is obtained and the results are exhibited graphically.

II. MATHEMATICAL FORMULATION

We consider a incompressible fluid-saturated horizontal porous layer of thickness d_m underlying a fluid layer of thickness d , the lower boundary of the porous layer is taken to be rigid (see Fig..1).

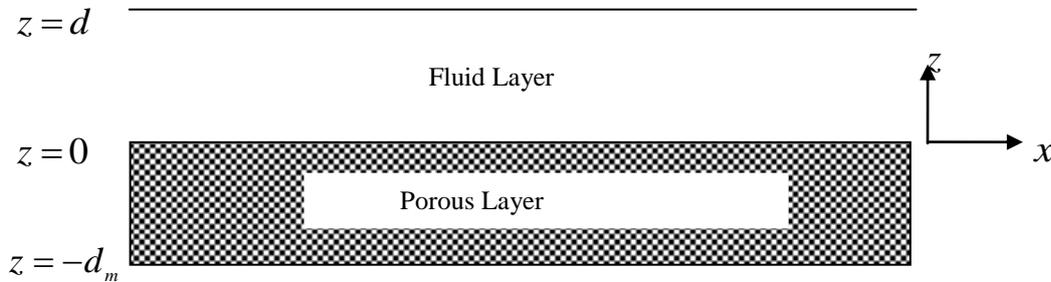


Fig. 1 Physical model

The lower hot rigid boundary $z_m = -d_m$, is kept at a constant temperature. While upper surface $z = d$ is free to atmosphere of constant temperature. A Cartesian coordinate system (x, y, z) is chosen such that the origin is at the interface between the fluid layer and the porous layer and the z -axis is vertically upward. The system is taken in a cavity to be large enough in horizontal extent, the xy -plane, and small in thickness in z -direction. It is also assumed that the ratio of the total height to the length of the layers is considerably small so that lateral end effects do not influence to the motions in the horizontal central part.

The governing equations for the fluid and the porous layers are:

$$\nabla \cdot \vec{V} = 0 \tag{1}$$

$$\rho_0 \left(\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} \right) = -\nabla p + \mu \nabla^2 \vec{V} \tag{2}$$

$$\frac{\partial T}{\partial t} + (\vec{V} \cdot \nabla) T = \kappa \nabla^2 T \tag{3}$$

$$\frac{\partial C}{\partial t} + (\vec{V} \cdot \nabla) C = \kappa_s \nabla^2 T \tag{4}$$

$$\nabla_m \cdot \vec{V}_m = 0 \tag{5}$$

$$\rho_0 \left(\frac{1}{\phi} \frac{\partial \vec{V}_m}{\partial t} + \frac{C_b}{\sqrt{K}} |\vec{V}_m| \vec{V}_m \right) = -\nabla_m p_m - \frac{\mu}{K} \vec{V}_m + \mu_m \nabla^2 \vec{V}_m \tag{6}$$

$$A \frac{\partial T_m}{\partial t} + (\vec{V}_m \cdot \nabla_m) T_m = \kappa_m \nabla_m^2 V_m \tag{7}$$

$$\Phi \frac{\partial C_m}{\partial t} + (\vec{V}_m \cdot \nabla_m) C_m = \kappa_{sm} \nabla_m^2 V_m \tag{8}$$

Here \vec{V} is the velocity vector, p is the pressure, T is the temperature, C is the concentration while \vec{V}_m , p_m , T_m , C_m are the corresponding quantities in the porous layer, κ is the thermal diffusivity, μ is the fluid viscosity,

g is the gravitational acceleration, ϕ is the porosity of the porous medium, A is the ratio of heat capacities, ρ_0 is the fluid density and K is the permeability of the porous medium.

Defining $\theta = T - T_b$, $\theta_m = T_m - T_{mb}$, $S = C - C_b$, $S_m = C_m - C_{mb}$, $p' = p - p_b$, $p_m' = p_m - p_{mb}$. In order to investigate the stability of the basic solution, infinitesimal disturbances are introduced and linearized in the usual manner. The pressure terms are eliminated by taking curl twice and only the vertical component is retained. We get the following perturbed equations

$$\frac{\partial}{\partial t}(\nabla^2 w) - \nu \nabla^4 w = 0 \tag{9}$$

$$\frac{\partial \theta}{\partial t} + w \frac{(T_u - T_0)}{d} = \kappa \nabla^2 \theta \tag{10}$$

$$\frac{\partial S}{\partial t} + w \frac{(C_u - C_0)}{d} = \kappa_s \nabla^2 \theta \tag{11}$$

$$\frac{1}{\Phi} \frac{\partial \nabla_m^2 w_m}{\partial t} - \nu_m \nabla_m^4 w_m - \frac{\nu}{\kappa} \nabla_m^2 w_m = 0 \tag{12}$$

$$A \frac{\partial \theta_m}{\partial t} + w_m \frac{(T_0 - T_L)}{d_m} = \kappa_m \nabla_m^2 \theta_m \tag{13}$$

$$\Phi \frac{\partial S_m}{\partial t} + w_m \frac{(C_0 - C_L)}{d_m} = \kappa_{sm} \nabla_m^2 S_m \tag{14}$$

Where $\nu = \mu / \rho$ and $\nu_m = \mu_m / \rho_m$ are kinematic viscosities of fluid and porous layers respectively.

$\nabla^2 = \nabla_h^2 + \partial^2 / \partial z^2$ is the Laplacian operator with $\nabla_h^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$. The corresponding quantities for the porous layer are $\nabla_m^2 = \nabla_{mh}^2 + \partial^2 / \partial z_m^2$ with $\nabla_{mh}^2 = \partial^2 / \partial x_m^2 + \partial^2 / \partial y_m^2$. Note that separate length scales are chosen for the two layers so that each layer is of unit depth so that both the layers of unit length

For the fluid layer

$$\vec{q} = \frac{\kappa}{d} \vec{q}', t = \frac{d^2}{\kappa} t', \theta = (T_0 - T_u) \theta', S = (C_0 - C_u) S', (x, y, z) = (x', y', z') \tag{14}$$

For the porous layer

$$\vec{q}_m = \frac{\kappa_m}{d_m} \vec{q}_m', t = \frac{d_m^2}{\kappa_m} t', \theta_m = (T_L - T_0) \theta_m', S_m = (C_L - C_0) S_m', (x_m, y_m, z_m) = (x_m', y_m', z_m' - 1) \tag{15}$$

and the non-dimensional disturbance equations are now given by

$$\left(\frac{1}{Pr} \frac{\partial}{\partial t} - \nabla^2 \right) \nabla^2 w = 0 \tag{16}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2 \right) \theta = w \tag{17}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2 \right) S = w \tag{18}$$

$$\left(\frac{Da}{Pr_m} \frac{\partial}{\partial t} + 1 \right) \nabla_m^2 w_m - \Lambda Da \nabla_m^4 w_m = 0 \quad (19)$$

$$\left(A \frac{\partial}{\partial t} - \nabla_m^2 \right) \theta_m = W_m \quad (20)$$

$$\left(\phi \frac{\partial}{\partial t} - \nabla_m^2 \right) S_m = W_m \quad (21)$$

where $Pr = \nu / \kappa$ is the Prantal number, $\tau = \kappa_s / \kappa$ is the ratio of diffusivity, $\tau = \mu / \rho_0$ is the kinematic viscosity of fluid layer, β is square root of Darcy number. The corresponding quantities for the porous layer are $Pr_m = \nu / \kappa_m$ is the Prantal number, $\tau_m = \kappa_{ms} / \kappa_m$ is the ratio of diffusivity, $\Lambda = \mu_m / \mu$ is the viscosity ratio.

Since the principle of exchange instability holds for surface tension driven convection either in fluid layer (see Pearson (1958) and Vidal (1966)) or in a porous layer (Hennenberg (1997), Rudraiah and Prasad (1998)), it is reasonable to assume that it holds good even for the present configuration as well. Further, the numerical calculations carried out for a wide range of parameters by Straughan (2001) also corroborates the validity of principle of exchange of stability for the superposed system. Hence, the time derivatives are dropped conveniently from Eqs. (16)-(21). Then performing a normal mode expansion of the dependent variables in both fluid and porous layers as

$$(w, T, C) = [W(z), \Theta(z), S(z)] f(x, y) \quad (22)$$

$$(w_m, T_m, C_m) = [W_m(z_m), \Theta_m(z_m), S_m(z_m)] f_m(x_m, y_m) \quad (23)$$

Where $f(x, y)$ and $f_m(x_m, y_m)$ are horizontal plan forms satisfying $\nabla_h^2 f = -a^2 f$ and $\nabla_{mh}^2 f_m = -a_m^2 f_m$. Here a and a_m are the horizontal wave numbers in fluid and porous layer respectively. For matching the solutions in two layers to be possible, we should have $a/d = a_m/d_m$, and hence $\zeta = a_m/a$, and substituting them in Eqs. (16)–(21) (with $\partial/\partial t = 0$), we obtain the following ordinary differential equations:

$$(D^2 - a^2)^2 W = 0 \quad (24)$$

$$(D^2 - a^2) \Theta = -f(z)W \quad (25)$$

$$(D^2 - a^2) S = -\frac{1}{\tau} W \quad (26)$$

$$\Lambda \beta^2 (D_m^2 - a_m^2 - 1) (D_m^2 - a_m^2) W_m = 0 \quad (27)$$

$$(D_m^2 - a_m^2) \Theta_m = -g(z_m)W_m \quad (28)$$

$$(D_m^2 - a_m^2) S_m = -\frac{1}{\tau_m} W_m \quad (29)$$

where D and D_m denote differentiation with respect to z and z_m respectively, $a = \sqrt{l^2 + m^2}$ and $a_m = \sqrt{\tilde{l}^2 + \tilde{m}^2}$ are correspondingly the overall horizontal wave numbers in the fluid and porous layers.

The boundary conditions are

$$W(1) = 0, \quad D^2W(1) + M_T a^2 \Theta(1) + M_S a^2 S(1) = 0 \quad (30)$$

$$D^3W(1) - 3a^2 DW(1) = 0, \quad D\Theta(1) = DS(1) = 0 \quad (31)$$

$$\hat{T}W(0) = W_m(1), \quad \hat{T}\zeta DW(0) = \Lambda D_m W(1) \quad (32)$$

$$\hat{T}\zeta D^2W(0) = \Lambda D_m^2 W(1), \quad \hat{T}\Theta_m(1) = \Theta(0) \quad (33)$$

$$\hat{S} S_m(1) = S(0), \quad D\Theta_m(1) = D\Theta(0), \quad DS_m(1) = DS(0) \quad (34)$$

$$\hat{T}\zeta^3 \beta^2 [D^3W(0) - 3a^2 DW(0)] = \Lambda \beta^2 [D_m^3 W_m(1) - 3a_m^2 D_m W_m(1)] - D_m W_m(1) \quad (35)$$

$$W_m(0) = D_m W_m(0) = D_m \Theta_m(0) = D_m S_m(0) = 0 \quad (36)$$

III. SOLUTION BY REGULAR PERTURBATION TECHNIQUE

For the assumed boundary conditions the eigen value problem is solved by using regular perturbation technique with wave number a as a perturbation parameter. The dependent variables in both the fluid and porous layers are now expanded in powers of a^2 in the form

$$(W, \Theta, S) = \sum_{i=0}^N (a^2)^i (W_i, \Theta_i, S_i) \quad (37)$$

$$(W_m, \Theta_m, S_m) = \sum_{i=0}^N (\zeta^2 a^2)^i (W_{mi}, \Theta_{mi}, S_{mi}) \quad (38)$$

Substitution of Eqs (37) and (38) into Eqs. (24)–(29) and the boundary conditions (30)–(36) yields a sequence of equations for the unknown functions $W_i(z), \Theta_i(z), S_i, W_{mi}(z), S_{mi}$ and $\Theta_{mi}(z)$ for $i = 0, 1, 2, 3, \dots$

At the leading order in a^2 Eqs. (24)–(29) become, respectively,

$$D^4W_0 = 0 \quad (39)$$

$$D^2\Theta_0 = -W_0 f(z) \quad (40)$$

$$D^2S_0 = -\frac{1}{\tau} W_0 \quad (41)$$

$$\Lambda \beta^2 D_m^4 W_{m0} - D_m^2 W_{m0} = 0 \quad (42)$$

$$D_m^2 \Theta_{m0} = -W_{m0} g(z) \quad (43)$$

$$D_m^2 S_{m0} = -\frac{1}{\tau_m} W_{m0} \quad (44)$$

and the boundary conditions (30)–(36) become

$$W_0(1) = 0, \quad D^2W_0(1) = 0 \quad (45)$$

$$D^3W_0(1) = 0, \quad D\Theta_0(1) = DS_0(1) = 0 \quad (46)$$

$$\hat{T}W_0(0) = W_{m0}(1), \quad \hat{T}\zeta DW_0(0) = \Lambda D_m W_{m0}(1) \quad (47)$$

$$\hat{T}\zeta D^2W_0(0) = \Lambda D_m^2W_{m0}(1), \quad \hat{T}\Theta_{m0}(1) = \Theta_0(0) \quad (48)$$

$$\hat{S}S_{m0}(1) = S_0(0), \quad D\Theta_{m0}(1) = D\Theta_0(0), \quad DS_{m0}(1) = DS_0(0) \quad (49)$$

$$\hat{T}\zeta^3\beta^2 D^3W_0(0) = \Lambda\beta^2 D_m^3W_{m0}(1) - D_mW_{m0}(1) \quad (50)$$

$$W_{m0}(0) = D_mW_{m0}(0) = D_m\Theta_{m0}(0) = D_mS_{m0}(0) = 0 \quad (51)$$

The solution to the zeroth order equations is given by

$$W_0 = 0, \quad \Theta_0 = \hat{T}, \quad S_0 = \hat{S} \quad (52)$$

$$W_{m0} = 0, \quad \Theta_{m0} = 1, \quad S_{m0} = 1. \quad (53)$$

At the first order in a^2 , Eqs (24)–(29) then reduce to

$$D^4W_1 = 0 \quad (54)$$

$$D^2\Theta_1 - \hat{T} = -W_1 f(z) \quad (55)$$

$$D^2S_1 - 1 = -\frac{1}{\tau}W_1 \quad (56)$$

$$\Lambda\beta^2 D_m^4W_{m1} - D_m^2W_{m1} = 0 \quad (57)$$

$$D_m^2\Theta_{m1} - 1 = -W_{m1}g(z_m) \quad (58)$$

$$D_m^2S_{m1} - 1 = -\frac{1}{\tau_m}W_{m1} \quad (59)$$

and the boundary conditions (30)–(36) become

$$W_0(1) = 0, \quad D^2W_0(1) = 0 \quad (60)$$

$$D^3W_0(1) = 0, \quad D\Theta_0(1) = DS_0(1) = 0 \quad (61)$$

$$\hat{T}W_0(0) = W_{m0}(1), \quad \hat{T}\zeta DW_0(0) = \Lambda D_mW_{m0}(1) \quad (62)$$

$$\hat{T}\zeta D^2W_0(0) = \Lambda D_m^2W_{m0}(1), \quad \hat{T}\Theta_{m0}(1) = \Theta_0(0) \quad (63)$$

$$\hat{S}S_{m0}(1) = S_0(0), \quad D\Theta_{m0}(1) = D\Theta_0(0), \quad DS_{m0}(1) = DS_0(0) \quad (64)$$

$$\hat{T}\zeta^3\beta^2 D^3W_0(0) = \Lambda\beta^2 D_m^3W_{m0}(1) - D_mW_{m0}(1) \quad (65)$$

$$W_{m0}(0) = D_mW_{m0}(0) = D_m\Theta_{m0}(0) = D_mS_{m0}(0) = 0 \quad (66)$$

The general solution of Eqs. (54) and (57) are respectively given by

$$W_1 = c_1 + c_2 z + c_3 z^2 + c_4 z^3 \quad (67)$$

$$W_{m1} = c_5 + c_6 z + c_7 e^{\sigma z} + c_8 e^{-\sigma z} \quad (68)$$

The constants $c_1 - c_8$ are determined using the boundary conditions and are found to be

$$c_1 = \frac{3 B_0 Da \varepsilon_T (2 \sqrt{Da} + \zeta \beta)}{\zeta^2 \Delta}, \quad c_2 = \frac{B_0 \varepsilon_T (-6 \sqrt{Da} - 3 \zeta^2 \beta + \zeta^2 / \sqrt{Da})}{\zeta^2 \Delta}$$

$$c_3 = \frac{6 B_0 \varepsilon_T \beta Da (\zeta^3 - \zeta - 1)}{2 \zeta \Delta}, \quad c_4 = \frac{B_0 \varepsilon_T \zeta (2 \sqrt{Da} + \zeta \beta)}{2 \Delta}, \quad c_5 = c_6 = \frac{-6 \varepsilon_T^2 (2 + \zeta \beta / \sqrt{Da})}{\zeta \Delta}$$

$$\Delta = 6 \zeta^2 \sqrt{Da} (B_0 - Cr M (1 + N S_m)) + (2 B_0 - 3 Cr M (1 + N S_m)) \zeta^3 \beta + 6 B_0 Da \zeta \beta (1 - \zeta)$$

Integrating Eqs. (55), (56), (58), (59) between $z = 0$ and 1 , using the relevant boundary conditions and adding the resulting equations, we obtain the following solvability condition:

$$\int_0^1 W_1 dz + \hat{d} \int_0^1 W_{m1} dz_m = \hat{T} + \hat{d}^2. \quad (69)$$

$$\tau_m \int_0^1 W_1 dz + \tau \hat{d} \int_0^1 W_{m1} dz_m = (\hat{S} + \hat{d}^2) \tau \tau_m. \quad (70)$$

$$\int_0^1 W_1 dz + \hat{d} \int_0^1 W_{m1} dz_m + \tau_m \int_0^1 W_1 dz + \tau \hat{d}^2 \int_0^1 W_{m1} dz_m = (\hat{S} + \hat{d}^2) \tau \tau_m + \hat{T} + \hat{d}^2 \quad (71)$$

Substituting for W_1 and W_{m1} in Eq.(71) and performing the integration, we obtain an expression for the critical Marangoni number. We note that for limiting case in $M_s = 0, f(z) = 1 = g(z_m)$ and as $\zeta \rightarrow 0, M_T \rightarrow 48$ this is the known exact value for a single fluid layer (Pearson(1958)).

IV. RESULTS AND DISCUSSION

The onset of double diffusive convection Marangoni convection in a composite layer consisting of a fluid layer overlying a porous layer. The eigen value problem is solved by regular perturbation technique with wave number α as a perturbation parameter and an analytical expression for the Marangoni number is obtained.

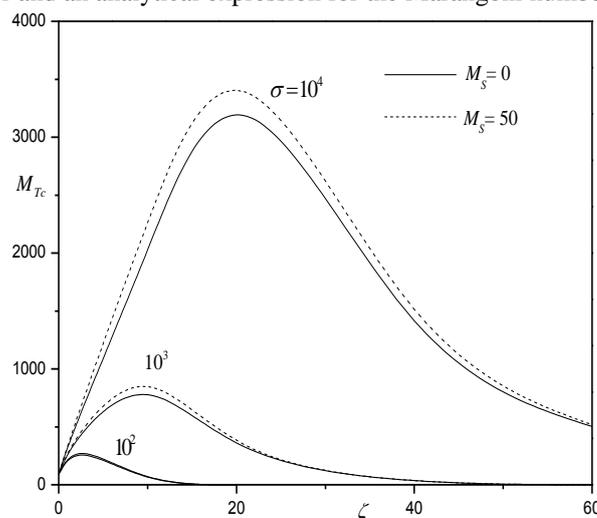


Fig 2. Variation of M_{Tc} as function of \hat{T} for different values of σ and M_s .

Figure 2 shows that the critical Marangoni number M_{Tc} as a function of ζ for linear temperature distribution. The effect of porous parameter σ for a set of fixed physical parameter $\tau = \tau_m = \hat{\kappa}_s = 1$ and $\Lambda = 10$ and different values of $\sigma = 10^2, 10^3$ and 10^4 . It is observed that the critical Marangoni number increases initially ζ and reaches a maximum and then decreases with further increase in ζ . This is because of $\sigma = d_m / \sqrt{\kappa}$, as ζ increases (i.e., $d_m \gg d$) the permeability parameter increases and hence a decrease in the critical Marangoni number. And also absorbed an increase in M_s is to increases the critical Marangoni number M_{Tc} and thus stabilise system.

Figure 3 shows that the critical Marangoni number M_{Tc} as a function of ζ for linear temperature distribution. The effect of viscosity ratio Λ for a set of fixed physical parameter $\tau = \tau_m = \hat{\kappa}_s = 1$ and $\sigma = 100$ and different values of $\Lambda = 0.5, 5$ and 10 . It is observed

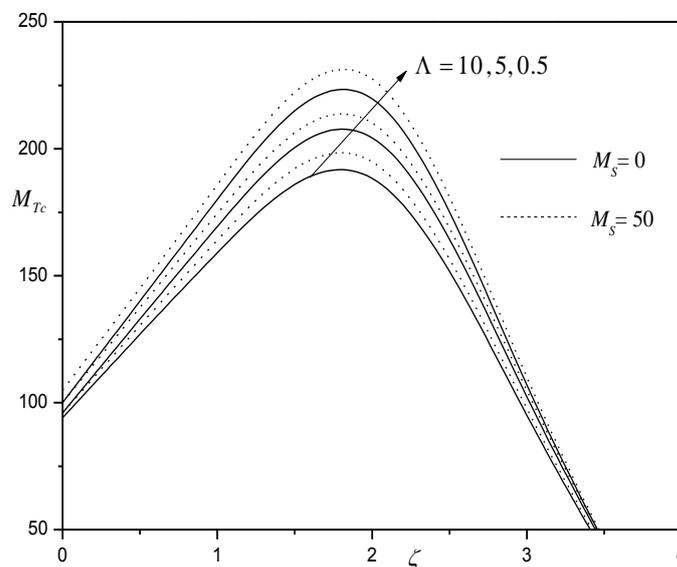


Fig. 3 Variation of M_{Tc} as function of \hat{T} for different values of Λ and M_s .

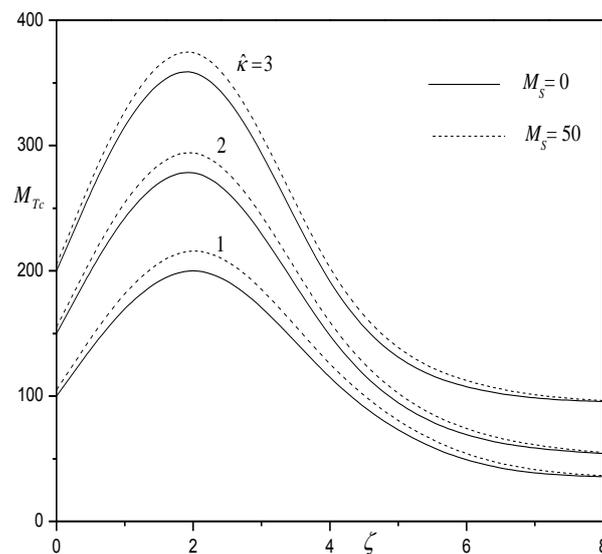


Fig. 4 Variation of M_{Tc} as function of \hat{T} for different values of $\hat{\kappa}$ and M_s .

that the critical Marangoni number increases initially ζ and reaches a maximum and then decreases with further increase in ζ . And also absorbed an increase in M_s is to increase the critical Marangoni number M_{Tc} and thus stabilise system.

The effect of thermal conductivity ratio $\hat{\kappa}$ for a set of fixed physical parameter $\tau = \tau_m = \hat{\kappa}_s = 1, \Lambda = 1$ and $\sigma = 100$ and different values of $\hat{\kappa} = 1, 2$ and 3 . From figure it is observed that the critical Marangoni number increases with $\hat{\kappa}$ and thus stabilise system.

V. CONCLUSIONS

The onset of double diffusive surface-tension-driven convection in two-layer system consisting of a fluid layer overlying a porous layer is analyzed theoretically.

The following conclusions may be made from this study.

1. When the porous parameter increases, the thermal critical Marangoni number increases and thus stabilize system.
2. The effect of viscosity ratio Λ on the onset of double diffusive Marangoni convections are more pronounced.
3. The critical thermal Marangoni number increase as the thermal conductivity ratio $\hat{\kappa}$ increases.

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