

# ON THE TERNARY QUADRATIC EQUATION

$$5(x^2 + y^2) - 9xy = 19z^2$$

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**Abstract:** The ternary quadratic Diophantine equation  $5(x^2 + y^2) - 9xy = 19z^2$  is analyzed for its patterns of non zero distinct integral solutions.

**Keywords:** Integral solutions, Ternary quadratic.

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## I. INTRODUCTION

The Ternary Quadratic Diophantine Equation offers an unlimited field for research because of their variety [1, 2]. For an extensive review of various problems, one may refer [3-23]. This communication concerns with yet another interesting Ternary Quadratic Equation  $5(x^2 + y^2) - 9xy = 19z^2$  for determining its infinitely many non-zero integral solutions.

## II. METHOD OF ANALYSIS

The ternary quadratic equation to be solved in integers is

$$5(x^2 + y^2) - 9xy = 19z^2 \quad (1)$$

It is noted that (1) can be satisfied by the following triples of integers  $(1141a, 379a, 419a), (-2294A, 1895A, 2095A), (151a, a, 77a), (746A, 5A, 385A), (305T, 94T, 115T), (-679A, 470A, 575A), (346a, 91a, 137a), (-911A, 455A, 685A), (1469A, 355A, 595A), (-166a, 71a, 119a), (481a, 70a, 215a), (-71T, 14T, 43T)$ .

Now, introducing the linear transformations

$$x = u + v \quad ; \quad y = u - v \quad (2)$$

in (1), it leads to

$$u^2 = 19(z^2 - v^2) \quad (3)$$

which is written as

$$\frac{u}{19(z+v)} = \frac{z-v}{u} = \frac{A}{B} \quad (4)$$

which is equivalent to the system of equations

$$Bu - 19Az - 19Av = 0$$

$$Au - Bz + Bv = 0$$

from which we get

$$u = 38AB \quad ; \quad v = B^2 - 19A^2 \quad ; \quad z = B^2 + 19A^2 \quad (5)$$

Using (5) in (2), we obtain the integer solutions to (1) as given below:

$$\begin{aligned} x &= B^2 - 38AB - 19A^2 \\ y &= 19A^2 + 38AB - B^2 \\ z &= B^2 + 19A^2 \end{aligned}$$

It is observed that, by rewriting (4) suitably, one may arrive at the following three patterns of solutions to (1).

**A. PATTERN : 1**

$$\begin{aligned} x &= 19B^2 + 38AB - A^2 \\ y &= A^2 + 38AB - 19B^2 \\ z &= A^2 + 19B^2 \end{aligned}$$

**B. PATTERN : 2**

$$\begin{aligned} x &= 19B^2 - 38AB - A^2 \\ y &= A^2 - 38AB - 19B^2 \\ z &= -A^2 - 19B^2 \end{aligned}$$

**C. PATTERN : 3**

$$\begin{aligned} x &= B^2 - 38AB - 19A^2 \\ y &= 19A^2 - 38AB - B^2 \\ z &= -19A^2 - B^2 \end{aligned}$$

However , we have another solution pattern which is illustrated below:

**D. PATTERN : 4**

(3) can be written as

$$u^2 + 19v^2 = 19 * z^2 \quad (6)$$

Assume  $z = a^2 + 19b^2$  (7)

Also 19 can be written as  $19 = (i\sqrt{19})(-i\sqrt{19})$  (8)

Substituting (7) and (8) in (6) and applying the method of factorization, define

$$u + i\sqrt{19}v = i\sqrt{19} * (a + i\sqrt{19}b)^2 \quad (9)$$

Equating the real and imaginary parts , we have

$$u = -38ab \quad ; \quad v = a^2 - 19b^2 \quad (10)$$

Using (10),(7) and (2), we obtain the integral solutions to (1) as presented below:

$$\begin{aligned} x &= a^2 - 38ab - 19b^2 \\ y &= 19b^2 - 38ab - a^2 \\ z &= a^2 + 19b^2 \end{aligned}$$

### III.CONCLUSION

To conclude, one may search for other patterns of solutions to the equation under consideration.

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