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COMPARATIVE RESPONSE OF DOUBLE HINGED ALP USING AIRY'S AND STOKES' WAVE THEORIES

Moazzam Aslam[1], Nazrul Islam[2], Mohd Moonis Zaheer[3], Mehtab Alam[2]

Research Scholar, Department of Civil Engineering, Jamia Millia Islamia, New Delhi-110025, India [1]

Professor, Department of Civil Engineering, Jamia Millia Islamia, New Delhi-110025, India [2]

Associate Professor, Civil Engineering Section, University Polytechnic, AMU Aligarh- 202002, India[3]

Abstract: Articulated loading platform (ALP) is one of the compliant offshore structures that are economically attractive especially as loading and mooring terminal in deep waters. These platforms are light in weight than conventional fixed platforms. An Articulated tower is a linear structure, flexibly connected to the sea bed through a universal joint and held vertically by the buoyant forces acting on it. The tower does not resist forces in bending due to wind, waves and currents rather; these forces are countered via a large buoyancy force. In this paper, dynamic analysis of the tower under regular waves has been carried out without current forces. The nonlinear governing equations of motion are derived using Lagrangian approach. Nonlinear effects due to variable submergence, buoyancy, added mass, instantaneous position of the tower and relative-velocity squared drag force are considered in the analysis. The equation of motion has been solved in time domain using NewMark's-β integration scheme. Modified Morison equation is used to model the fluid forces as these equations account for non-linearities associated with vortex shedding effects accurately in comparison to standard Morison equation. Analytical studies are conducted to compare the response of double hinged articulated tower under regular waves using Airy's wave theory evaluated with Chakrabarti's modification and that obtained by using Stokes' fifth order nonlinear wave theory. Stokes fifth order non-linear theory agrees closely in deep and intermediate water and it is found that for higher waves the difference in the values of responses obtained by Airy's and Stokes' are lesser while the difference is significantly higher for smaller waves. Results show that the deck displacement response as well as hinge rotation and hinge shear obtained using Stokes' theory are lesser than that obtained using the Airy's theory.

Keywords: Offshore structures; Dynamic analysis; Waves; Morison equation; Stokes' theory

I. INTRODUCTION

The term compliance can be defined as "degree of yielding. under applied force". "A compliant offshore structure is a structure in the marine environment that accommodates the (dynamic) forces by flexibility instead of resisting the loads rigidly, thereby limiting the internal (dynamic) loads." Articulated towers belongs to the group of compliant offshore structure which have been found quite attractive and suitable for deep water applications. In other words, an articulated loading platform is a compliant offshore structure which is connected to the sea bed through a universal joint. The evaluation of hydrodynamic forces due to waves on the structural members of ALP is important for its economic and safe design. Halvacioglu and Incecik (1990) studied the dynamic response of single and double hinged articulated tower subjected to wave and wind forces. They conduct their studies to predict the response of tower due to change in position of buoyancy chamber, hinge location and weight of deck paltform and concluded that due to change in position of buoyancy chamber, a significant change in natural frequency of tower was observed. Kim and Ran (1994) presented the responses of an articulated loading platform in random waves and currents both in frequency and time domain. They concluded from their numerical examples that slowly varying resonant responses in random waves are significant compared to wave frequency responses in case of no current or current normal to the wave direction. However, a great reduction observed when there exists strong in-line (coplanar or adverse) current. Islam et al. (2009 and 2012) studied the responses of single and double hinged articulated towers and compare under various ocean environments. Langrangian approach was used in deriving the non linear equations of motion. Pierson Moskowitz spectrum has been used in the characterization of sea state and Simiu's spectrum has been used for the estimation of fluctuating wind. They concluded that wind effects are an important factor in determining the survivability of double hinged articulated towers in harsh offshore environments. Murtedjo et al. (2005) presented the study regarding the effects of buoyancy variations towards the dynamic behaviour of articulated tower for both regular and random wave. The authors found that by +/- 20% change in the outside diameter of the tower shaft ,the natural frequency get affected by as much as +/- 27.3% . They also found that by increasing the outside diameter by 20% the exciting moment energy will get increases by 28.2% and by decreasing the outside diameter by 20% the exciting moment energy will get decreased by 22.4%.

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Variations in the buoyancy chamber length gave the same effect but that is smaller than varying the outside diameter. The authors concluded that Articulated towers are very feasible to be operated at extreme wave condition because of their relatively small natural frequency and maximum response amplitude operator (RAO). Nagamani and Ganapathy (2000) studied a three legged articulated tower using analytical and experimental techniques. The authors also presented the effects of mass distribution on the variations of bending moment and the deck accelerations. The model was tested in a 2m flume for various wave frequencies and wave heights of regular waves. The authors concluded that the maximum bending moment along the legs increases with the wave frequency and decreases with the natural frequency of the tower also the bending moment increases with wave height for all the three legs. They further concluded that the deck acceleration increases with wave height and decreases with the natural frequency of the tower. Chandrasekaran et al. (2007) presented the response behaviour of TLP under regular waves using Stokes theory by considering the coupling between various degrees of freedom. They considererd the various nonlinearities developed due to change in tether tension, change in buoyancy and hydrodynamic drag force. They performed studies under regular waves using both Airy and Stokes theory. They concluded that the coupled response in surge and pitch degree of freedom obtained using Stokes theory is lesser than that obtained by the Airy theory.
Since there exist many nonlinearities in the sea itself and all the studies available in the literature on ALP are with the use of Airy's linear wave theory, so there is a need to study the behaviour of ALP by the use of nonlinear wave theory to have more accurate results. Therefore, our aim is to study the ALP responses with the use of Stokes fifth order non linear wave theory under regular wave and compare them with Airy's theory. It is observed that resuts obtained from Stokes theory are significantly lesser than that given by Airy's theory.

II. PROBLEM DESCRIPTION

In the present study, a double hinged articulated loading platform (ALP) is modeled as an inverted double pendulum comprised of two universal hinges as shown in Fig. 1. It consist of a ballast chamber attached to the lower shaft of length L_1 which is attached to the sea bed by a universal/articulation joint. The upper portion consists of a buoyancy chamber attached to upper column of length L_2 which is connected to the lower shaft by another universal joint. The in-plane rotations at the two articulation points constitute the dynamic degree of freedom of the system. The system has two genralized coordinates; rotations θ_1 and θ_2 about the vertical axes. Following considerations are made in the structural modelling.

- A buoyancy force F, keeps the pendulum in a stable upright position.
- Fluid added mass is directly included in the inertia forces.
- Fluid inertia forces due to fluid acceleration and drag forces proportional to the square of the relative velocity between the fluid and the shaft are considered.
- Effect of collinear current on the water particle kinematics is considered.

The equation of motion for the double hinged articulated tower under regular wave is given below:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \tag{1}$$

where [M] is the mass matrix consisting of structural mass and added mass moment of inertia, [C] is the damping matrix and [K] is stiffness matrix. $\{\ddot{x}\}$, $\{\dot{x}\}$ and $\{x\}$ are the vectors for structural acceleration, velocity and displacement respectively. {F(t)} is the forcing function at any instant of time due to waves consisting of both drag and inertia forces. Drag and inertia forces are calculated by using Morison's equation.

A. *MASS MATRIX, M*

The mass matrix of the ALP is presented below:

$$M = \begin{bmatrix} I_{1t} + \overline{m_{2t}} L_1^2 + m_d L_1^2 & \overline{m_{2t}} c_2 L_1 \cos(\theta_2 - \theta_1) \\ \overline{m_{2t}} c_2 L_1 \cos(\theta_2 - \theta_1) & I_{2t} + I_d + m_d L_p^2 \end{bmatrix} \tag{2}$$

where $\overline{m_{2t}}$ is the total mass of upper tower evaluated as

$(m_2 + m_a)$ and m_2 is structural mass of upper tower.
$m_a = m_{ac} + m_{af}$ is the added mass of the structure. Where m_{ac} is the time invariant added mass up to MSL and m_{af} is the fluctuating added mass which depends upon the variable submergence of the structure with respect to MSL with the passage of waves.

B. DAMPING MATRIX, C

The Coulomb damping matrix involves with the square term of velocity is as follows:

$$C = \begin{bmatrix} 0 & -\overline{m_{2t}}\, c_2 L_1 \sin(\theta_2 - \theta_1)\dot{\theta}_2 \\ \overline{m_{2t}}\, c_2 L_1 \sin(\theta_2 - \theta_1)\dot{\theta}_1 & 0 \end{bmatrix} \tag{3}$$

C. STIFFNESS MATRIX, K

The stiffness matrix K of the ALP is

$$K = \begin{vmatrix} \{(F_1 b_1 - W_1 c_1) + (F_2 - W_2 - W_d)L_1\}\frac{\sin\theta_1}{\theta_1} & 0 \\ 0 & (F_2 b_2 - W_2 c_2 - W_d L_P)\frac{\sin\theta_2}{\theta_2} \end{vmatrix} \tag{4}$$

III. WAVE THEORIES

A. *AIRY'S THEORY*

A relatively simple theory of wave motion known as Airy's linear theory has been given by G.B.Airy in 1842 (Dawson,1983). The theory assumes a sinusoidal wave form whose height *H* is small in comparison with the wavelength *L* and the water depth *d*.

As per the Airy's theory the sea surface elevation (η) at given x and t is

$$\eta(x,t) = \frac{H}{2}\cos(kx - \omega t) \tag{5}$$

Where k =2π/L and ω= 2π/T

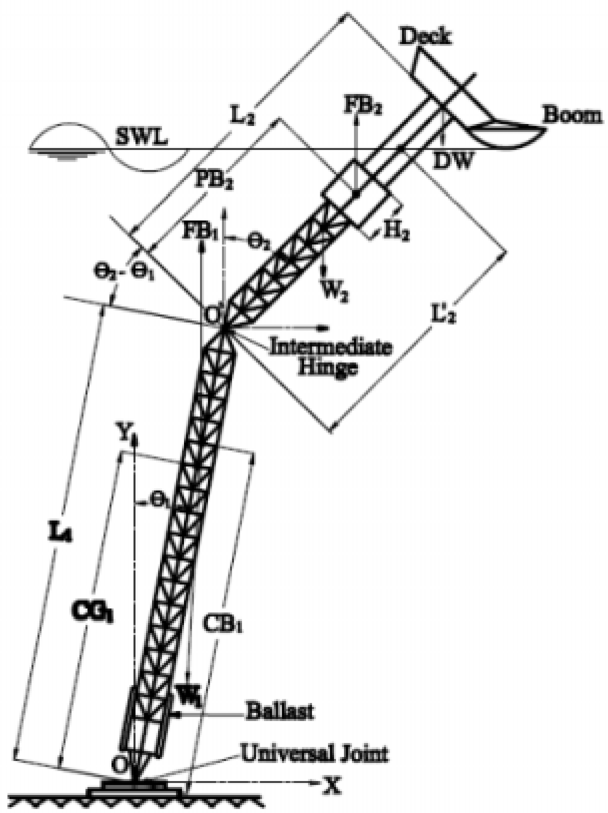

Fig. 1. Double hinged articulated loading platform

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The horizontal and vertical components of water particle velocities are as given below:

$$\dot{u}\,(x,t) = \frac{\partial \phi}{\partial x} = \frac{\omega H}{2}\,\frac{\cos h\big(k(d+z)\big)}{\sinh(kd)}\cos(kx - wt) \tag{6}$$

$$\dot{v}\,(x,t) = \frac{\partial \phi}{\partial z} = \frac{\omega H}{2}\,\frac{\sin h(k(d+z))}{\sinh(kd)}\,\sin\,(kx - wt) \tag{7}$$

Here we assume that at ground level z = - d and at SWL z = 0, so the velocities are at SWL

$$\dot{u}(x,t) = \frac{\partial \phi}{\partial x} = \frac{\omega H}{2}\frac{\cosh(kd)}{\sinh(kd)}\cos\,(kx - \omega t) \tag{8}$$

$$\dot{v}(x,t) = \frac{\partial \phi}{\partial z} = \frac{\omega H}{2}\frac{\sinh(kd)}{\sinh(kd)}\sin\,(kx - \omega t) \tag{9}$$

The horizontal and vertical components of water particle acceleration at SWL are as given below:

$$\ddot{u}(x,t) = \frac{\partial \dot{u}}{\partial t} = \frac{\omega^2 H}{2}\frac{\cosh(kd)}{\sinh(kd)}\sin\,(kx - \omega t) \tag{10}$$

$$\ddot{v}(x,t) = \frac{\partial \dot{v}}{\partial t} = -\frac{\omega^2 H}{2}\frac{\sinh(kd)}{\sinh(kd)}\cos\,(kx - \omega t) \tag{11}$$

B. STOKES' FIFTH ORDER NONLINEAR WAVE THEORY

Using perturbation approach, higher number of terms in the series of the non-linear theory is considered. Stokes assumed that all variation in the X direction can be represented by Fourier series and that the coefficients in these series can be written as perturbation expansions in terms of a parameter which increases with wave height. Because of the slowness of the convergence in the series of shallow water, the theory is considered to be valid in the regime where d/L is greater than 0.1.

According to Stokes fifth order nonlinear wave theory, the instantaneous vertical displacement of sea surface above the SWL is given as (Dawson, 1983)

$$\eta(x,t) = \frac{1}{k}\sum_{n=1}^{5} F_n\ \cos\ n(kx - wt) \tag{12}$$

The horizontal and vertical components of water particle velocities are as given below:

$$\dot{u}\,(x,t) = \frac{\omega}{k}\sum_{n=1}^{5} G_n\frac{\cosh(nkz)}{\sinh(nkd)}\cos\ n(kx - wt) \tag{13}$$

$$\dot{v}\,(x,t) = \frac{\omega}{k}\sum_{n=1}^{5} G_n\frac{\sinh(nkz)}{\sinh(nkd)}\sin n(kx - wt) \tag{14}$$

The horizontal and vertical particle acceleration can be determined based on the following expression:

$$\ddot{u}(x,t) = \frac{k c_s^2}{2}\sum_{n=1}^{5} R_n \sin n(kx - wt) \tag{15}$$

$$\ddot{v}(x,t) = \frac{-k c_s^2}{2}\sum_{n=1}^{5} S_n \cos\ n(kx - wt) \tag{16}$$

The wave speed c_s is given by

$$c_s = \left[\frac{g}{k}\,(1 + a^2 C_1 + a^4 C_2)\tanh kd\right]^{1/2} \tag{17}$$

IV. NUMERICAL STUDY

A double hinged articulated loading platform in 420 m water depth has been considered for the numerical study. The idealized tower consists of two segment vertical cantilever having a lumped mass at the top. Each cantilever is discretized in 50 elements. The characteristics of the platform and the environment used in the present study are same as used by "Islam, N., Zaheer, M.M. and Ahmad, S. (2009)". The natural frequencies of the system for the two modes

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of vibration are 0.14 rad/s and 0.42 rad/s respectively. The dynamic response of double hinged articulated loading platform is obtained under regular wave without current forces with the use of Airy linear wave theory as well as the Stokes nonlinear wave theory. Two sea states are hereby considered for (H = 10m, T = 10s) and (H = 15m, T = 15s). The sea is simulated for the duration of one hour. It is important to mention here that simulated length excludes the initial transient non stationary phase of the responses due to initial conditions.

V. Simulation results

The response of deck displacement, lower and upper hinge rotation, base and upper hinge shear for ALP under 10 m/10 s waves without current velocity is plotted. Fig. 2 show the response of deck displacement; Figs. 3-4 show the response of lower and upper hinge rotation; Figs. 5-6 show the response of base and upper hinge shear.

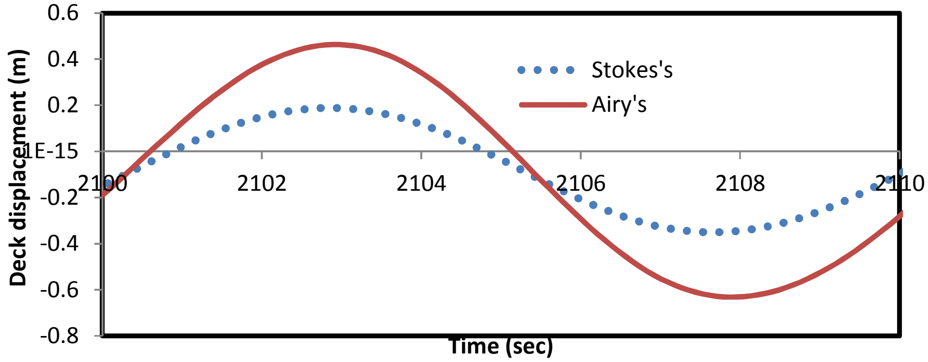

Fig. 2. Deck displacement response of ALP (10 m/10 s)

It is seen from the plotted graphs that the maximum positive response obtained using Stokes' fifth order nonlinear wave theory is less than those obtained from Airy's linear wave theory with the Chakrabarti's modification.

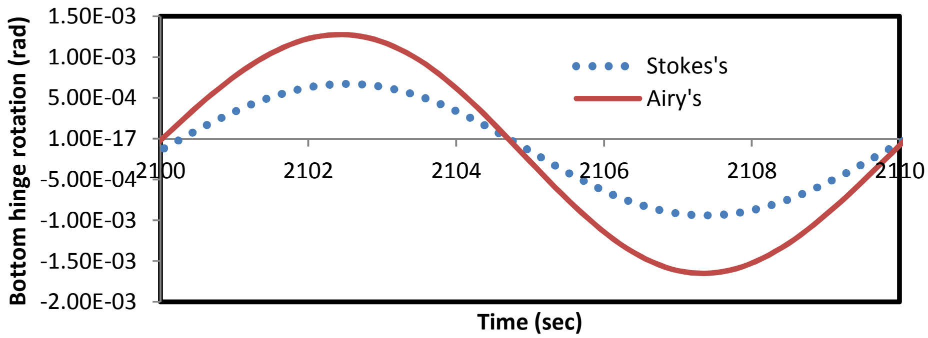

Fig. 3. Bottom hinge rotation response of ALP (10 m/10 s)

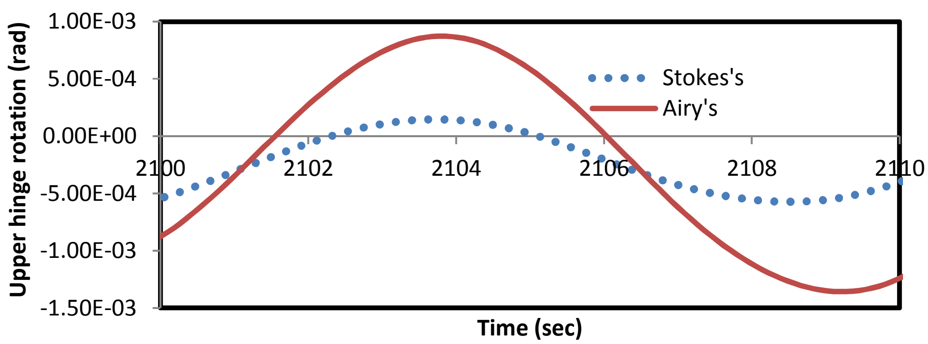

Fig. 4. Upper hinge rotation response of ALP (10 m/10 s)

The Table 1 shows the comparative statistical response obtained by both the theories in terms of mean, standard deviation, maximum and minimum values for 10m/10s waves without current forces.

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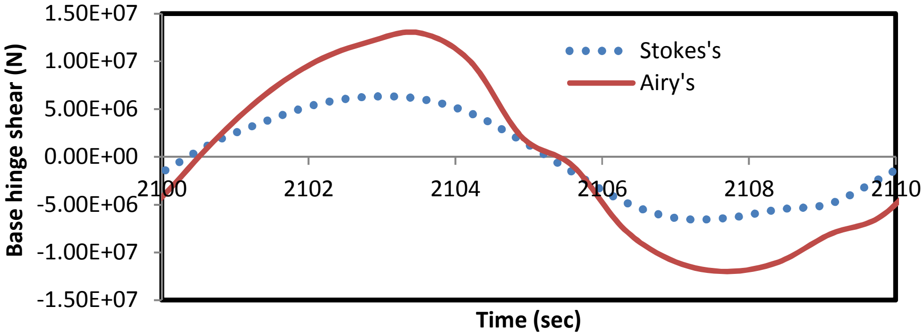

Fig. 5. Base hinge shear response of ALP (10 m/10 s)

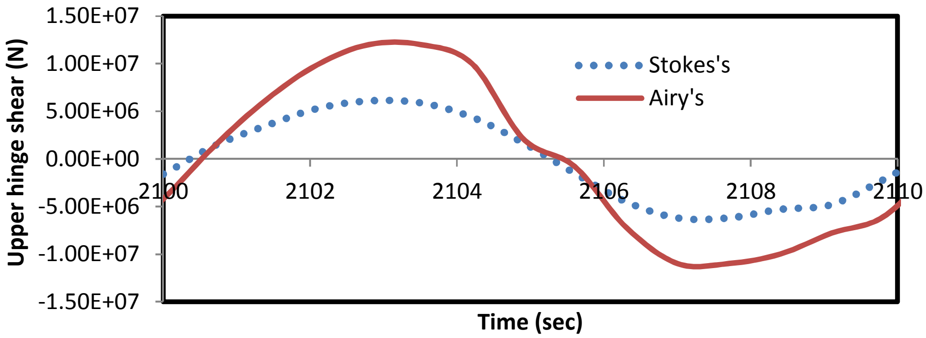

Fig. 6. Upper hinge shear response of ALP (10 m/10 s)

The response of deck displacement, lower hinge rotation, upper hinge rotation, base hinge shear and upper hinge shear for ALP under 15 m/15 s waves without current velocity is plotted. Fig. 7 show the response of deck displacement; Figs. 8-9 show the response of lower and upper hinge rotation; Figs. 10-11 show the response of base and upper hinge shear.

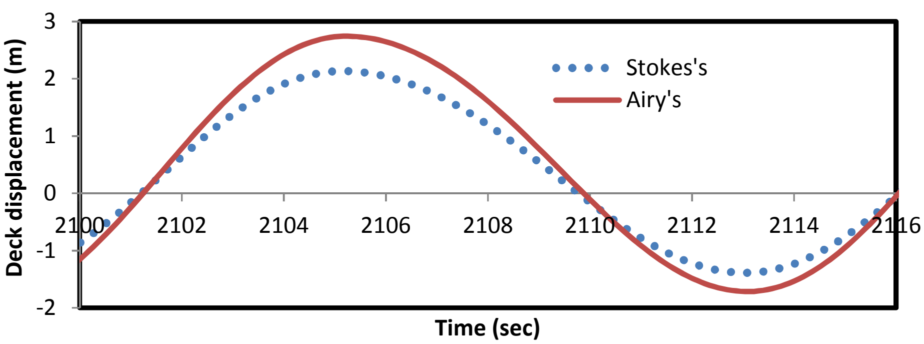

Fig. 7. Deck displacement response of ALP (15 m/15 s)

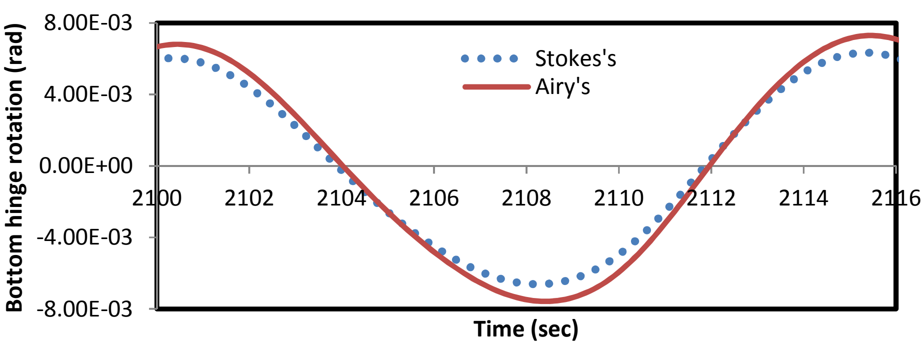

Fig. 8. Bottom hinge rotation response of ALP (15 m/15 s)

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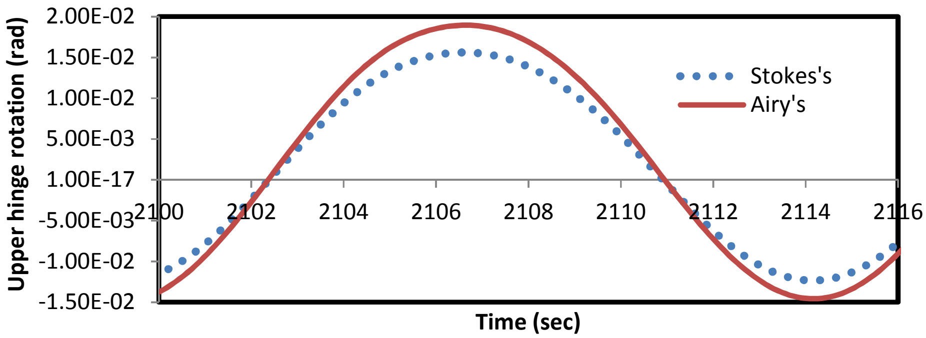

Fig. 9. Upper hinge rotation response of ALP (15 m/15 s)

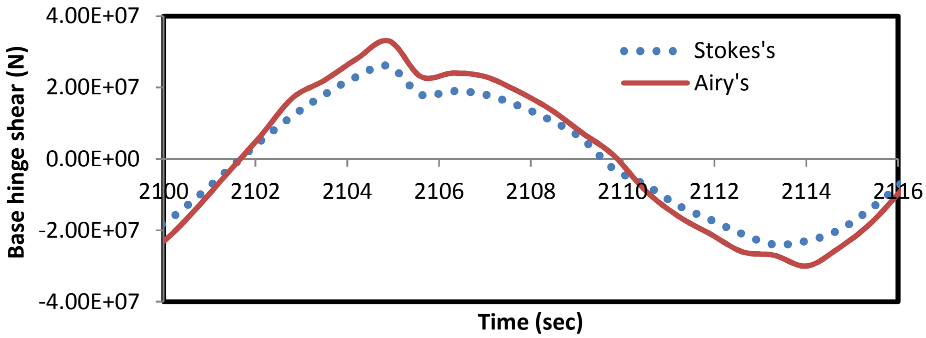

Fig. 10. Base hinge shear response of ALP (15 m/15 s)

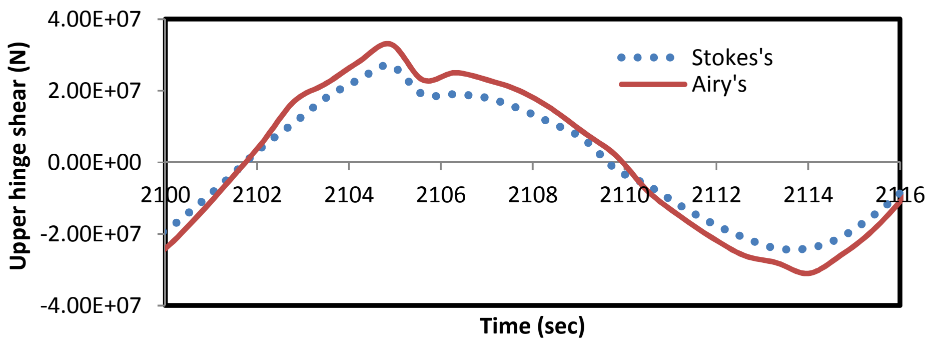

Fig. 11. Upper hinge shear response of ALP (15 m/15 s)

The Table 2 shows the comparative statistical response obtained by both the theories in terms of mean, standard deviation, maximum and minimum values for 15m/15s waves without current forces.

VI. CONCLUSIONS

Based on the performed analytical studies, the following conclusions are drawn:

1. The response obtained using Stokes's nonlinear wave theory for the hydrodynamic case of 10m/10s waves without current forces are lesser by 57.7%, 46.8%, 65.6%, 50% and 47.5% for deck displacement, bottom hinge rotation, upper hinge rotation, base hinge shear and upper hinge shear respectively in comparision to that obtained by using Airy's linear wave theory with Chakrabarti's modifications.
2. The response obtained using Stokes's nonlinear wave theory for the hydrodynamic case of 15m/15s waves without current forces are lesser by 20.9%, 13.2%, 16.4%, 22.2% and 22.2% for deck displacement, bottom hinge rotation, upper hinge rotation, base hinge shear and upper hinge shear respectively in comparision to that obtained by using Airy's linear wave theory with Chakrabarti's modifications.

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3. For the same water depth, it is seen that the responses under 10m/10s waves using stokes' theory gives about 54% lesser values than that obtained by Airy's wave theory, while for 15m/15s waves, the stokes' theory gives about 19% lesser values than that obtained by Airy's wave theory.
4. For higher waves the difference in the values of responses obtained by Airy's and stokes' are lesser (19%) while the difference is significantly higher (54%) for smaller waves.

Table 1
Comparative Statistical response by Airy's and Stokes theory for ALP under sea state (H_s = 10.0 m, T_z = 10 sec) without current

Response	Mean		S.D		Maximum		Minimum	
	Airy	Stokes	Airy	Stokes	Airy	Stokes	Airy	Stokes
Deck displacement (m)	-0.0399	-0.0578	0.3774	0.2032	0.621	0.263	- 0.632	- 0.394
Bottom hinge rotation (rad)	- 9.18 x 10^{-5}	- 9.21 x 10^{-5}	1.05 x 10^{-3}	5.92 x 10^{-4}	1.55 x 10^{-3}	8.24 x 10^{-4}	-1.73 x 10^{-3}	- 9.88 x 10^{-4}
Upper hinge rotation (rad)	-7.64 x 10^{-5}	-1.61 x 10^{-4}	7.05 x 10^{-4}	2.98 x 10^{-4}	1.19 x 10^{-3}	4.09 x 10^{-4}	- 1.36 x 10^{-3}	- 7.02 x 10^{-3}
Base hinge shear (N)	- 7.88 x 10^{3}	- 8.40 x 10^{4}	8.78 x 10^{6}	4.67 x 10^{6}	1.30 x 10^{7}	6.50 x 10^{6}	- 1.20 x 10^{7}	- 6.60 x 10^{6}
Upper hinge shear (N)	- 1.98 x 10^{4}	- 9.23 x 10^{4}	8.42 x 10^{6}	4.48 x 10^{6}	1.20 x 10^{7}	6.30 x 10^{6}	- 1.10 x 10^{7}	- 6.40 x 10^{6}

Table 2
Comparative Statistical response by Airy's and Stokes theory for ALP under sea state (H_s = 15.0 m, T_z = 15 sec) without current

Response	Mean		S.D		Maximum		Minimum	
	Airy	Stokes	Airy	Stokes	Airy	Stokes	Airy	Stokes
Deck displacement (m)	0.520	0.407	1.655	1.278	2.921	2.312	- 2.038	- 1.588
Bottom hinge rotation (rad)	-3.72 x 10^{-4}	-3.78 x10^{-4}	5.07 x 10^{-3}	4.51 x 10^{-3}	7.49 x 10^{-3}	6.50 x 10^{-3}	-7.57 x 10^{-3}	- 6.82 x 10^{-3}
Upper hinge rotation (rad)	2.94 x 10^{-3}	2.41 x 10^{-3}	1.21 x 10^{-2}	1.00 x 10^{-2}	1.95 x 10^{-2}	1.63 x 10^{-2}	-1.55 x 10^{-2}	-1.27 x 10^{-2}
Base hinge shear (N)	1.66 x 10^{6}	1.06 x 10^{6}	2.10 x 10^{7}	1.66 x 10^{7}	3.60 x 10^{7}	2.80 x 10^{7}	- 3.20 x 10^{7}	- 2.50 x 10^{7}
Upper hinge shear (N)	1.60 x 10^{6}	1.01 x 10^{6}	2.13 x 10^{7}	1.67 x 10^{7}	3.60 x 10^{7}	2.80 x 10^{7}	- 3.30 x 10^{7}	- 2.50 x 10^{7}

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