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Thermo-Diffusion and Chemical Reaction Effects on a Steady Mixed Convective Heat and Mass Transfer Flow with Induced Magnetic Field

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Abstract: This paper is focuses on the effects of soret and chemical reaction on a two dimensional, laminar, mixed convective heat and mass transfer flow of a viscous incompressible electrically conducting and radiating fluid over a vertical porous plate in the presence of a transverse applied magnetic field and constant suction, by taking induced magnetic field into account. The boundary layer equations have been transformed into dimensionless coupled non-linear ordinary differential equations by similarity transformations. The resultant dimensionless governing equations are solved by series solution technique. The influences of various thermo physical parameters on the velocity, temperature, concentration and induced magnetic fields are discussed. It is observed that the induced magnetic field increases with a rise in chemical reaction parameter or Hartmann number or radiation parameter or Schmidt number, whereas it decreases with an increase in the soret number. This model finds applications in laminar magneto-aero dynamics, material process and MHD propulsion thermo fluid dynamics.

Keywords: Mixed convection, Magneto hydrodynamics, Heat and mass transfer, Soret effect, chemical reaction, induced magnetic field, perturbation technique, current density.

I.INTRODUCTION

Several investigators have made model studies on the effect of magnetic field on free convection flows. Some of them are Geogantopoulos [1], Nanousis et al. [2] and Raptis and Singh [3]. Raptis and Soundalgekar [4] investigated the steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in presence of heat source/sink. The combined heat and mass transfer of an electrically conducting fluid in MHD natural convection adjacent to a vertical surface was analyzed by chen [5]. The study of heat and Mass transfer on the free convective oscillatory flow of a viscous incompressible fluid past an infinite vertical porous plate in presence of transverse sinusoidal suction velocity and a constant free stream velocity was presented by Ahmed [6].

The growing need for chemical reactions in chemical and hydrometallurgical industries requires the study of heat and mass transfer with chemical reaction. Dekha *et al.* [7] investigated the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past a vertical plate with a constant heat and mass transfer. Chamkha [8] studied the MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation/ absorption and a chemical reaction. Muthucumaraswamy [9] presented heat and mass transfer effects on a continuously moving isothermal vertical surface with uniform suction by taking into account the homogeneous chemical reaction of first order.



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In view of the importance of this diffusion – thermo effect, similarity equations of the momentum energy and concentration equations are derived by introducing a time dependent length scale. Seddeek [10] has considered the influence of variable viscosity on mixed free-forced convective flow and mass transfer over an accelerating surface in the presence of Soret, Dufour and heat source effects. Combined chemical reaction and Soret/Dufour effects on free convection heat and mass transfer in Darcian porous media were studied also very recently by Postelnicu [11].

In all the above mentioned studies, the magnetic Reynolds number is assumed to be very small, therefore the induced magnetic field is neglected. To accurately predict the flow behavior, such effect must be considered for larger values of magnetic Reynolds number. Sahin Ahmed [12], investigated the influence of thermal radiation and magnetic prandtl number on the stady MHD heat and mass transfer by mixed convective flow of a gray gas, taking into account the induced magnetic field. A numerical study of the natural convection heat and mass transfer about a vertical surface embedded in a saturated porous medium under the influence of a magnetic field has been done by Postelnicu [13], taking into account the diffusion-thermo and thermal-diffusion effects.

However, the simultaneous effects of Soret and chemical reaction in the presence of induced magnetic field have not been studied. Hence an attempt is made to analyse the effect of Soret and chemical reaction on a MHD steady mixed convective heat and mass transfer flow over a semi-infinite vertical porous plate taking into account the induced magnetic field. The governing equations are solved employing a regular perturbation technique.

II. MATHEMATICAL ANALYSIS

A two-dimensional mixed convective heat and mass transfer flow of a viscous, incompressible, electrically conducting fluid past a continuously moving semi-infinite vertical porous plate under the influence of a transversely applied magnetic field is considered. It is assumed that:

- 1) The fluid considered is an optically thin gray gas.
- 2) All the fluid properties except the density in the buoyancy force term are constant.
- 3) The Eckert number, Ec is small.
- 4) The magnetic Reynolds number of the flow is not taken to be small enough so that the induced magnetic field is not negligible.
- 5) The wall is maintained at constant temperature $\overline{T_w}$ and concentration $\overline{C_w}$ higher than the ambient temperature $\overline{T_{\infty}}$ and concentration $\overline{C_{\infty}}$ respectively.
- 6) The suction velocity is taken to be $\bar{v} = -v_0$.

Following the Cartesian coordinate system, the flow is assumed to be in the \overline{X} -direction, which is taken along the vertical plate in the upward direction, where \overline{Y} -direction normal to the plate. Since the plate is infinite in length in \overline{X} -direction, therefore, all the physical quantities except possibly the pressure are assumed to be independent of \overline{X} .Let $\overline{q} = (\overline{U}(y), \overline{V}, 0)$ be the fluid velocity and $\overline{H} = (\overline{H_x}(y), \overline{H_y}, 0)$ be the magnetic induction vector at a point $(\overline{X}, \overline{Y}, \overline{Z})$ in the fluid. The schematic diagram of the flow configuration and coordinate system of the problem are shown in Fig. 1





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Within the frame of such assumptions and under the Oberbeck- Boussinesq's approximation and in consistency with boundary layer theory, the governing equations relevant to the problem are

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$-\nu_0 \frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(\bar{T} - \overline{T_\infty}) + g\bar{\beta}(\bar{c} - \overline{c_\infty}) + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\mu_0 H_0}{\rho} \frac{\partial \overline{H_x}}{\partial \bar{y}}$$
(2)

$$-\nu_0 \frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho c_p} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial \bar{y}}$$
(3)

$$-\nu_0 \ \frac{\partial \overline{H_x}}{\partial \overline{y}} = \frac{1}{\sigma \mu_0} \ \frac{\partial^2 \overline{H_x}}{\partial \overline{y}^2} + H_0 \ \frac{\partial \overline{u}}{\partial \overline{y}}$$
(4)

$$-\nu_0 \frac{\partial \bar{c}}{\partial \bar{y}} = D \frac{\partial^2 \bar{c}}{\partial \bar{y}^2} - \bar{k} \left(\bar{c} - \overline{c_{\infty}} \right) + D_T \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}$$
(5)

The relevant boundary conditions are

$$\bar{u}=0, \bar{T} = \overline{T_w}, \overline{H_x} = 0, \bar{C} = \overline{C_w} \quad \text{at} \quad \bar{y} = 0$$

$$\bar{u} \to U_0, \bar{T} \to \overline{T_\infty}, \overline{H_x} \to 0, \bar{C} \to \overline{C_\infty} \quad \text{as} \quad \bar{y} \to \infty$$
(6)

where g is the acceleration due to gravity, β is the coefficient of thermal expansion, $\overline{\beta}$ is the concentration expansion coefficient, v is the kinematic viscosity of the fluid, μ_0 is the magnetic permeability, H_0 is the applied constant magnetic field, H_x is induced magnetic field, ρ is the density of the fluid, σ is the electrical conductivity, κ is thermal conductivity, D is the chemical molecular diffusivity, c_p is the specific heat capacity of the fluid at constant pressure and \overline{k} is chemical reaction parameter.

The non-dimensional quantities are

$$y = \frac{v_0 \bar{y}}{v}, u = \frac{\bar{u}}{U_0}, \theta = \frac{\bar{T} - \overline{T_{\infty}}}{\overline{T_w} - \overline{T_{\infty}}}, \phi = \frac{\bar{C} - \overline{C_{\infty}}}{\overline{C_w} - \overline{C_{\infty}}}, Sc = \frac{v}{D}, Pr = \frac{\rho v c_p}{\kappa}, Gr = \frac{v g \beta (\overline{T_w} - \overline{T_{\infty}})}{U_0 v_0^2}, Gm = \frac{v g \bar{\beta} (\overline{C_w} - \overline{C_{\infty}})}{U_0 v_0^2},$$

$$Pr_m = \sigma v \mu_0, M = \sqrt{\frac{\mu_0}{\rho}} \frac{H_0}{v_0}, H = \sqrt{\frac{\mu_0}{\rho}} \frac{H_x}{v_0}, R = \frac{64 a v \bar{\sigma} \overline{T_{\infty}}^3}{\rho v_0^2 c_p}, K = \frac{\bar{K}v}{v_0^2}, S_0 = \frac{D_T (\overline{T_w} - \overline{T_{\infty}})}{v(\overline{C_w} - \overline{C_{\infty}})}$$

$$(7)$$

For the case of an optically thin gray gas, the local radiation absorption is expressed as

$$\frac{\partial q_r}{\partial \bar{y}} = -4a\bar{\sigma} \left(\bar{T}^4_{\infty} - \bar{T}^4\right) \tag{8}$$

where 'a' is the mean absorption coefficient and $\overline{\sigma}$ is the Stefan-Boltzmann constant. It is assumed that the temperature differences within the flow are sufficiently small such that \overline{T}^4 may be expressed as linear function of the temperature \overline{T} . This is accomplished by expanding \overline{T}^4 in Taylor series about \overline{T}_{∞} and neglecting higher order terms, thus

$$\bar{T}^4 \cong 4\bar{T}^3_{\infty}\bar{T} - 3\bar{T}^4 \tag{9}$$

Using the transformations (6) and with the help of (7),(8) the non-dimensional forms of (1)-(4) are

$$\frac{d^2u}{dy^2} + \frac{du}{dy} + M\frac{dH}{dy} + \theta Gr + \phi Gm = 0$$
⁽¹⁰⁾

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$$\frac{d^2\theta}{dy^2} + Pr\frac{d\theta}{dy} - \frac{PrR}{4} = 0 \tag{11}$$

$$\frac{d^2H}{dy^2} + Pr_m \frac{dH}{dy} + Mpr_m \frac{du}{dy} = 0$$
(12)

$$-\frac{d\phi}{dy} = \frac{1}{Sc}\frac{d^2\phi}{dy^2} - K\phi + S_0\frac{d^2\theta}{dy^2}$$
(13)

The corresponding boundary conditions are

$$u = 0, \theta = 1, H = 0, \phi = 1 \quad \text{at} \quad y = 0$$

$$u \to 1, \theta \to 0, H \to 0, \phi \to 0 \quad \text{as} \quad y \to \infty$$
(14)

III. SOLUTION OF THE PROBLEM

Solving equations (11) and (13) subject to the boundary conditions (14) we get

$$\theta = e^{-\xi y} , \quad \phi = A_1 e^{-\xi y} + A_2 e^{-\eta y} \tag{15}$$

Now, in order to solve the coupled equations (10) and (12) under the boundary conditions (14), it is assumed that the solutions of the equations to be of the forms

$$R(y) = R_0(y) + Ec R_1(y) + O(Ec^2)$$
(16)

which R denotes a general dependent variable u, H and Ec is the Eckert number, which is very small for an incompressible fluid.

Substituting (16) in (10) and (12) and equating the coefficients of the same degree terms and neglecting terms of O (Ec^2) , the following ordinary differential equations are obtained.

$$u_0'' + u_0' + MH_0' + Gr\theta + Gm\phi = 0 \tag{17}$$

$$u_1'' + u_1' + MH_1' = 0 (18)$$

$$H_0^{''} + M p r_m u_0^{'} + p r_m H_0^{'} = 0 (19)$$

$$H_1'' + M p r_m u_1' + p r_m H_1' = 0 (20)$$

The corresponding boundary conditions are

$$u_{0} = 0, u_{1} = 0, H_{0} = 0, H_{1} = 0 \text{ at } y = 0$$

$$u_{0} \to 1, u_{1} \to 1, H_{0} \to 0, H_{1} \to 0 \text{ as } y \to \infty$$
 (21)

Solutions of the velocity and induced magnetic field subject to the boundary conditions (21) are

$$u(y) = 1 + A_3 e^{-\xi y} + A_4 e^{-\eta y} + A_5 e^{-\lambda y}$$
(22)

$$H(y) = A_6 e^{-\lambda y} + A_7 e^{-\xi y} + A_8 e^{-\eta y} + A_9 e^{-pr_m y}$$
(23)



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The constants are not given here to save space.

The skin-friction, Nusselt numbr and Sherwood number are important physical parameters for this type of boundary layer flow.

Knowing the velocity field, the skin-friction at the plate can be obtained, which in non-dimensional form is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -(\xi A_3 + \eta A_4 + \lambda A_5)$$
(24)

Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in non-dimensional form, in terms of the Nusselt number, is given by

$$Nu = \left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -\xi \tag{25}$$

Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in non-dimensional form, in terms of Sherwood number, is given by

$$sh = \left(\frac{\partial\phi}{\partial y}\right)_{y=0} = -(\xi A_1 + \eta A_2) \tag{26}$$

The non dimensional current density at the plate y=0 is given by

$$J = \left(\frac{\partial H}{\partial y}\right)_{y=0} = -(\xi A_7 + \eta A_8 + \lambda A_6 + pr_m A_9)$$
⁽²⁷⁾

IV. RESULTS AND DISCUSSION

Final results are composed for the main physical parameters which are presented by means of graphs. The influence of the chemical reaction parameter (k), soret number (S_o), Magnetic prandtl number (pr_m), radiation parameter (R) on the velocity, temperature, concentration and induced magnetic field profiles can be analyzed from Fig. 2-11.

Fig.2 illustrate the variation in velocity distribution across the boundary layer for various values of the chemical reaction parameter (k). it can be seen that the velocity decreases with the increase of k. it is noticed from the graph that the velocity at the start of the boundary layer increases slowly till it attains the maximum value, after that they are decreasing and this trend is seen for all the values of the parameter k.





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Fig.3. depicts the effect of soret number (S_o) on the fluid velocity and we observed as increase in the fluid velocity as soret number (S_o) increases. This is because an increase in the volumetric rate of generation increases the buoyancy force there y increasing fluid velocity. The effect of magnetic prandtl number (pr_m) on the velocity field is shown in Fig. 4. It is clear that an increase in pr_m causes an increase in velocity profile.



Fig. 5. Effect of R on temperature.



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Fig. 5. show the effect of radiation parameter R on the temperature profile. A rise in R causes a significant fall in the temperature values from the highest value at the wall (y=0) across the boundary layer to the free stream. Thus, greater values of R corresponds to smaller radiation flux and the minimum temperature observed. Radiation thereby reduces the rate of energy transport to the fluid. The variation of temperature profile for different values if the prandtl number (pr) is shown in Fig.6. The results show that an increase of the prandtl number results in a decrease in the thermal boundary layer thickness and a more uniform temperature distribution across the boundary layer. The reason is that the smaller value of Pr are equivalent to increasing the thermal conductivities and therefore, heat is able to diffuse away from the heated surface more rapidly than for the larger values of Pr. Hence, the thicker the boundary layer is the slower the rate of heat transfer.



Fig. 8. Effect of S_o on concentration.



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Fig.7.show the effect of chemical reaction parameter (K) on concentration profiles. It is noticed that species concentration and thermal boundary layer are decreasing, as the values of chemical reaction increasing. The influence of soret number (S_o) on the concentration of the fluid is seen in Fig. 8. In general it is noted that increase in soret number contributes to increase in concentration of the medium. The effect of magnetic prandtl number (Pr_m) on the induced magnetic field (H) is presented in the Fig.9. In this Fig. magnetic prandtl number (Pr_m) is set as less than unity, which implies that the magnetic diffusion rate exceeds the viscous diffusion rate. As such Pr_m increases, momentum diffusivity will be increased. Therefore, when Pr_m increases from 0.1 to 0.4, the induced magnetic field is found to increase absolutely in the boundary layer $0 \le y \le 2.5$, but this trend is opposite for the region $2.5 < y \le 10$.



Fig. 11.Effect of K and S₀ on skin friction.



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The effect of soret number (S_o) on the induced magnetic field is illustrated in Fig. 10. It is observed from the Fig. that, an increase in the soret number leads to a decrease in the induced magnetic field. The effect of chemical reaction parameter (K) and soret number (S_o) on the skin friction (τ) against Hartmann number (M) has been presented in the Fig. 11. Investigation shows that, the skin friction increases in magnitude with the increase of both K as well as S_o .

Tables 1 and 2 represents a comparison of the numerical values of the velocity and induced magnetic field obtained in the present case with that of Sahin Ahmed [12] for different values of radiation parameter (R). It is clear that there is an excellent agreement.

TABLE I: COMPARISON OF THE VELOCITY (*U*) OF PRESENT CASE WITH THAT OF SAHIN AHMED [12] FOR DIFFERENT VALUES OF R, AND *y*=10, *M*=0.25, *Prm*=0.1, *Sc*=0.6, *K*=0.5, *Gr*=5.0, *Gm*=5.0, *Pr*=0.71, *S_o*=1.0.

	Sahin Ahmed[12]			Present work		
У	R=0.1	R=0.8	R=1.0	R=0.1	R=0.8	R=1.0
2.0	1.1302	1.0922	1.0861	1.0836	1.0368	1.0290
4.0	1.0901	1.0732	1.0707	1.0632	1.0398	1.0362
6.0	1.0324	1.0268	1.0261	1.0208	1.0122	1.0110
8.0	1.0101	1.0085	1.0083	1.0057	1.0030	1.0026
10.0	1.0030	1.0026	1.0025	1.0014	1.0006	1.0006

TABLE II: COMPARISON OF THE INDUCED MAGNETIC FIELD (*H*) OF PRESENT CASE WITH THAT OF SAHIN AHMED [12] FOR DIFFERENT VALUES OF *R*, AND y=10, *M*=0.25, *Prm*=0.1, *Sc*=0.6, *K*=0.5, *Gr*=5.0, *Gr*=5.0, *Pr*=0.71, *Sc*=1.0.

	Sahin Ahmed[12]			Present work		
У	R=0.1	R=0.8	R=1.0	R=0.1	R=0.8	R=1.0
2.0	0.0144	0.0150	0.0151	0.0151	0.0158	0.0159
4.0	0.0111	0.0121	0.0122	0.0123	0.0135	0.0137
6.0	0.0082	0.0092	0.0094	0.0096	0.0108	0.0111
8.0	0.0064	0.0073	0.0074	0.0077	0.0088	0.0090
10.0	0.0052	0.0059	0.0060	0.0062	0.0072	0.0073

V.CONCLUSION

An analysis of the two dimensional steady laminar mixed convective heat and mass transfer of a viscous incompressible electrically conducting fluid over a vertical porous plate is presented under the action of a transverse applied magnetic field. The thermo-diffusion (Soret) effect and chemical reaction is taken into consideration. The transformed system of non-linear, coupled, ordinary differential equations governing the problem were solved analytically by using perturbation technique. A comprehensive set of graphical results for the velocity, temperature, concentration and induced magnetic field is presented and their dependence on some physical parameters is discussed. The observations are:

- An increase in S_o (Soret number) leads to a decrease in the values of induced magnetic field. However, there is an increase in both the velocity and concentration with an increase in S_o .
- An increase in K (chemical reaction parameter) increases the induced magnetic field where as it decreases the velocity and concentration.
- Temperature decreases with an increase in the Prandtl number (Pr) or radiation parameter (R).
- An increase in Hartmann number (M) decreases the velocity but increases the induced magnetic field.
- Current density rises with the magnetic Prandtl number (*Prm*).
- The dimensionless skin-friction coefficient is found to increase with increasing values of S_o or K.



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