

Further Results on Complementary Super Edge Magic Graph Labeling

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Abstract : In this paper we introduced the concept of complementary super edge magic labeling and Complementary Super Edge Magic strength of a graph G. A graph $G(V, E)$ is said to be complementary super edge magic if there exist a bijection $f: V \cup E \rightarrow \{1, 2, \dots, p+q\}$ such that $p+q+1 - f(x)$ is constant. Such a labeling is called complementary super edge magic labeling with complementary super edge magic strength. In this paper for a graph $G(V, E)$ the complementary super edge magic labeling and minimum of all constants which is called complementary super edge magic strength of G is defined. In this paper, we investigate whether some families of graphs are complementary or not?

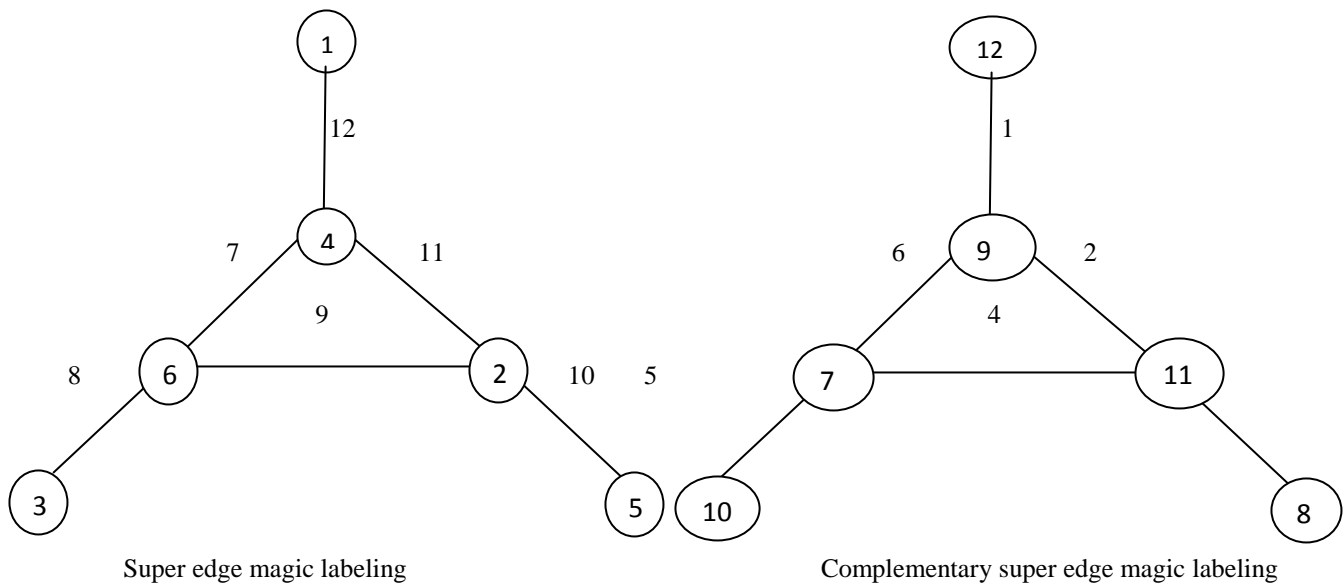
Keyword: Graph Labeling, Edge Magic Labeling, Total Edge Magic Labeling, Super Edge Magic Labeling, Complementary Super Edge Magic Labeling.

I. INTRODUCTION

A labeling of a graph G is an assignment of mathematical objects to vertices, edges, or both vertices and edges subject to certain conditions. Graph labeling have applications in coding theory, networking addressing, and in many other fields. In most applications the labels are positive (or nonnegative) integers. In 1963, Sedlack introduced a new class of labeling called magic labeling for a graph $G(V, E)$, which is defined as a bijection f from E to a set of positive integers such that

- (I) $f(e_i) \neq f(e_j) \quad \forall$ distinct e_i and e_j ,
- (II) $\sum_{e \in N(x)} f(e)$ is same for every $x \in V$, where $N(x)$ is the set of edges incident to x.

In 1970 Kotzig and Rosa defined another labeling which is called total edge magic labeling. A graph $G(V, E)$ with p vertices and q edges is called total edge magic if there is a bijection $f: V \cup E \rightarrow \{1, 2, \dots, p+q\}$ such that there exist a constant k for any edge $uv \in E$, we have $f(u) + f(uv) + f(v) = k$. Kotzig and Rosa proved that all cycles, complete bipartite graphs and caterpillar are total edge magic. A complete graph K_n is total edge magic if and only if $n \in \{1, 2, 3, 5, 6\}$. Total edge magic graph is called super edge magic if $f(V(G)) = \{1, 2, \dots, p\}$. Given an super edge –magic labeling f of a graph $G(p, q)$, the function $\bar{f}(x)$ such that $\bar{f}(x) = p+q+1-f(x)$ for all $x \in G$ is said to be complementary super edge –magic labeling to f(x). Two super edge magic f_1 and f_2 of G are equivalent if $f_1 = f_2$ or $f_1 = \bar{f}_2$. An super edge magic f of G is said to be self complimentary super edge magic if $f = \bar{f}$. Super edge magic strength of a graph $G(V, E)$ is denoted $sms(G)$ and it is defined as the minimum of all constants where the minimum is taken over all edge magic labeling of G. For super edge magic strength of a graph $G(V, E)$ complementary super edge magic strength is defined and it is denoted by $csmgs(G)$ i.e. $csmgs(G) = \min\{\bar{k} : \bar{f}$ is complementary super edge strength of G}. Fig 1 represents graph with super edge magic labeling and complementary super edge magic labeling.



II. MAIN RESULTS

In this paper the complementary super edge magic labeling and csems of two well known graphs such as the generalized prism $C_m \times P_n$ and $G \cong T(n, n, n-1, n, 2n-1)$ are obtained.

Before giving our main results we give a necessary and sufficient conditions for some graphs to have complementary super edge magic labeling.

- I. Every cycle C_n has super edge-magic labeling if and only if n is odd .
- II. $K_{m,n}$ has complementary super edge-magic labeling if and only if $m = 1$ or $n = 1$.
- III. K_n has complementary super edge-magic labeling if and only if $n = 1, 2,$ or 3 .
- IV. nK_2 has complementary super edge-magic labeling if and only if n is odd .
- V. $2P_n$ has complementary super edge-magic labeling if and only if $n \neq 2$ or 3 .
- VI. The friendship graph consisting of n triangles has complementary super edge-magic labeling if and only if $n = 3, 4, 5$ or 7 .
- VII. nP_3 has complementary super edge-magic labeling for $n = 4$ and n is even .
- VIII. $P_{2n(+)} N_m$ is a graph with $2n+m$ vertices and $2(m+n)-1$ number of edges admits complementary super edge magic labeling for all $n, m \geq 1$.

Theorem. 1 The generalized prism $C_m \times P_n$ is super edge magic for every odd m and $n \geq 2$ with $sems = \frac{6mn - m + 3}{2}$ and complementary super edge magic strength $= \frac{12mn - 5m + 3}{2}$.

Proof. Let $G \cong C_m \times P_n$ be the generalized prism with $V(G) = \{ v_{i,t} : 1 \leq i \leq m, 1 \leq t \leq n \}$

And

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$$E(G) = \{ v_{i,t} v_{i+1,t} : 1 \leq i \leq m, 1 \leq t \leq n \} \cup \{ v_{i,t} v_{i,t+1} : 1 \leq i \leq m, 1 \leq t \leq n-1 \}$$

Consider the function

$$f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, mn, mn+1, \dots, 3mn-m\}$$

Defined as

$$f(v_{i,t}) = \begin{cases} \frac{i+1}{2} & \text{if } 1 \leq i \leq m \text{ is odd and } t = 1, \\ \frac{j+m+1}{2} & \text{if } 1 \leq j \leq m \text{ is even and } t = 1, \\ \frac{i+m(2t-2)}{2} & \text{if } 1 \leq i \leq m \text{ is even and } 2 \leq t \leq n, \\ \frac{j+m(2t-1)}{2} & \text{if } 1 \leq j \leq m \text{ is odd and } 2 \leq t \leq n. \end{cases}$$

and

$$f(E(G)) = \begin{cases} \frac{6mn-2m+1-i-j}{2}, & 1 \leq i, j \leq m, i \text{ is even and } j \text{ is odd with } t = 1, \\ \frac{6mn-4mt+2m-i-j+3}{2}, & 1 \leq i, j \leq m, i \text{ is even and } j \text{ is odd with } 2 \leq t \leq n. \end{cases}$$

Thus f is a super edge-magic of prism constant $\frac{6mn-m+3}{2}G$ with magic and complementary super edge-magic labeling \bar{f} is defined as

$$\bar{f}: V(G) \cup E(G) \rightarrow \{1, 2, \dots, mn, \dots, 3mn-m\}$$

$$\bar{f}(V(G)) = \begin{cases} \frac{6mn-2m+1-i}{2} & \text{if } 1 \leq i \leq m \text{ is odd and } t = 1, \\ \frac{6mn-3m-j+1}{2} & \text{if } 1 \leq j \leq m \text{ is even and } t = 1, \\ \frac{6mn-i-2mt+2}{2} & \text{if } 1 \leq i \leq m \text{ is even and } 2 \leq t \leq n, \\ \frac{6mn-m-j-2mt+2}{2} & \text{if } 1 \leq j \leq m \text{ is odd and } 2 \leq t \leq n. \end{cases}$$

$$\bar{f}(E(G)) = \begin{cases} \frac{i+j+1}{2} & \text{if } 1 \leq i, j \leq m, i \text{ is even and } j \text{ is odd with } t = 1 \\ \frac{4mt-4m+i+j-1}{2} & \text{if } 1 \leq i, j \leq m, i \text{ is even and } j \text{ is odd with } 2 \leq t \leq n. \end{cases}$$

Thus $\bar{f}(G)$ is complementary super edge -magic labeling of f with magic constant $\frac{12mn-5m+3}{2}$ i.e. $csems = \frac{12mn-5m+3}{2}$.

Theorem 2. The graph $G \cong T(n, n, n-1, n, 2n-1)$ is super edge magic for odd $n \geq 3$ with $sems = 15n$ and complementary super edge magic labeling for any odd $n \geq 3$ with $csems = 21n - 6$.

Proof. Let vertices and edges of G as follows

$$V(G) = \{c\} \cup \{x_{ij} : 1 \leq i \leq 5; 1 \leq j \leq n_i\},$$

$$E(G) = \{cx_i : 1 \leq i \leq 5\} \cup \{x_{ij} x_{i,j+1} : 1 \leq i \leq 5; 1 \leq j \leq n_i - 1\}.$$

Here $p=6n-1$, and $q = 6n-2$. Consider the function defined as :

$$f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 12n-3\}.$$

$$f(c) = 4n+1,$$

and for odd j ,

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$$f(u) = \begin{cases} \frac{j+1}{2}, & \text{for } u = x_{1j} \\ n+1 - \frac{j-1}{2}, & \text{for } u = x_{2j} \\ \frac{j+1}{2} + n+1, & \text{for } u = x_{3j} \\ \frac{n-j}{2} + \frac{3(n+1)}{2}, & \text{for } u = x_{4j} \\ \frac{2n-j-1}{2} + 2(n+1), & \text{for } u = x_{5j} \end{cases}$$

for even j

$$f(u) = \begin{cases} \frac{j}{2} + 3n+1, & \text{for } u = x_{1j} \\ n-j - \frac{1}{2} + 3n+1 + \frac{n+1}{2}, & \text{for } u = x_{2j} \\ \frac{j}{2} + 4n+1, & \text{for } u = x_{3j} \\ \frac{n-j-1}{2} + 4n+1 + \frac{n+1}{2}, & \text{for } u = x_{4j} \\ \frac{2n-j-2}{2} + 5n+1, & \text{for } u = x_{5j} \end{cases}$$

and f(E(G)) is defined by

$$f(uv) = \begin{cases} 12n - \frac{(i+j+3)}{2}, & \text{for } u = x_{1j} \\ 10n + \frac{i+j-5}{2}, & \text{for } u = x_{2j} \\ 10n - \frac{(i+j+5)}{2}, & \text{for } u = x_{3j} \\ 8n + \frac{i+j-5}{2}, & \text{for } u = x_{4j} \\ 6n + \frac{i+j-3}{2}, & \text{for } u = x_{5j} \end{cases}$$

Thus f is a super edge-magic of G with constant 15n, G and complementary super edge-magic labeling \bar{f} is defined as for odd j,

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$$\bar{f}(u) = \begin{cases} 12n - \frac{j+5}{2}, & \text{for } u = x_{1j} \\ 11n + \frac{j-7}{2}, & \text{for } u = x_{2j} \\ 11n - \frac{j+7}{2}, & \text{for } u = x_{3j} \\ 10n + \frac{j-7}{2}, & \text{for } u = x_{4j} \\ 9n + \frac{j-7}{2}, & \text{for } u = x_{5j} \end{cases}$$

for even j,

$$\bar{f}(u) = \begin{cases} 9n - \frac{j+6}{2}, & \text{for } u = x_{1j} \\ 8n + \frac{j-6}{2}, & \text{for } u = x_{2j} \\ 8n - \frac{j+4}{2}, & \text{for } u = x_{3j} \\ 7n + \frac{j-4}{2}, & \text{for } u = x_{4j} \\ 6n + \frac{j-2}{2}, & \text{for } u = x_{5j} \end{cases}$$

and $\bar{f}(E(G))$ is defined by

$$\bar{f}(v) = \begin{cases} \frac{i+j-1}{2}, & \text{for } v = x_{1j}x_{1j+1} \\ 2n - \frac{i+j-1}{2}, & \text{for } v = x_{2j}x_{2j+1} \\ 2n + \frac{i+j+1}{2}, & \text{for } v = x_{3j}x_{3j+1} \\ 4n - \frac{i+j-1}{2}, & \text{for } v = x_{4j}x_{4j+1} \\ 6n - \frac{i+j+1}{2}, & \text{for } v = x_{5j}x_{5j+1} \end{cases}$$

Similarly we can defined $\bar{f}(cx_i)$ for $1 \leq i \leq 5$.

Thus $\bar{f}(G)$ is complementary super edge -magic labeling of f with magic constant $21n-6$ i.e. $csems = 21n-6$.
With this paper, we hope that interest in super edge-magic. and complementary super edge-magic labeling will aroused among those who study graph labeling.

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